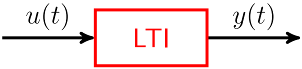
Vrije Universiteit Brussel

Block-oriented modeling

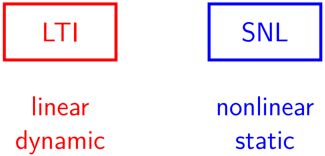
Maarten Schoukens   Koen Tiels

Introduction

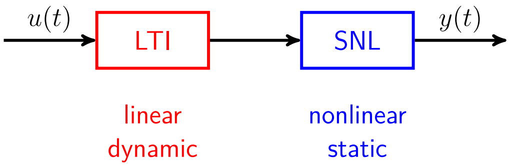


linear
dynamic

Block-oriented models consist of linear dynamics and static nonlinearities

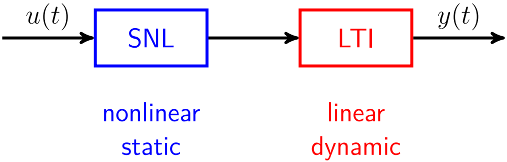


Block-oriented models consist of linear dynamics and static nonlinearities



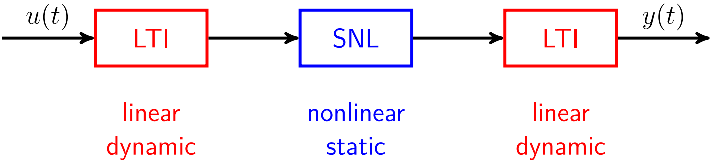
- Wiener

Block-oriented models consist of linear dynamics and static nonlinearities



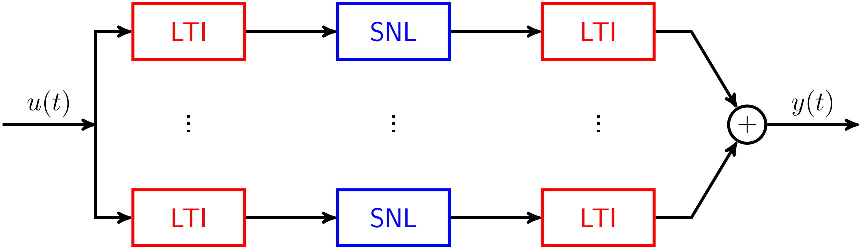
- Wiener
- Hammerstein

Block-oriented models consist of linear dynamics and static nonlinearities



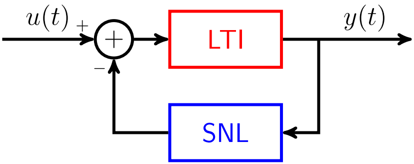
- Wiener
- Hammerstein
- Wiener-Hammerstein

Block-oriented models consist of linear dynamics and static nonlinearities



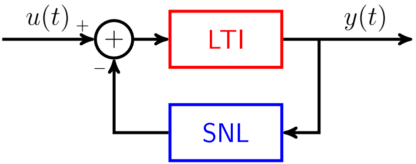
- Wiener
- Hammerstein
- Wiener-Hammerstein
- parallel Wiener-Hammerstein

Block-oriented models consist of linear dynamics and static nonlinearities



- Wiener
- Hammerstein
- Wiener-Hammerstein
- parallel Wiener-Hammerstein
- nonlinear feedback

Block-oriented models consist of linear dynamics and static nonlinearities

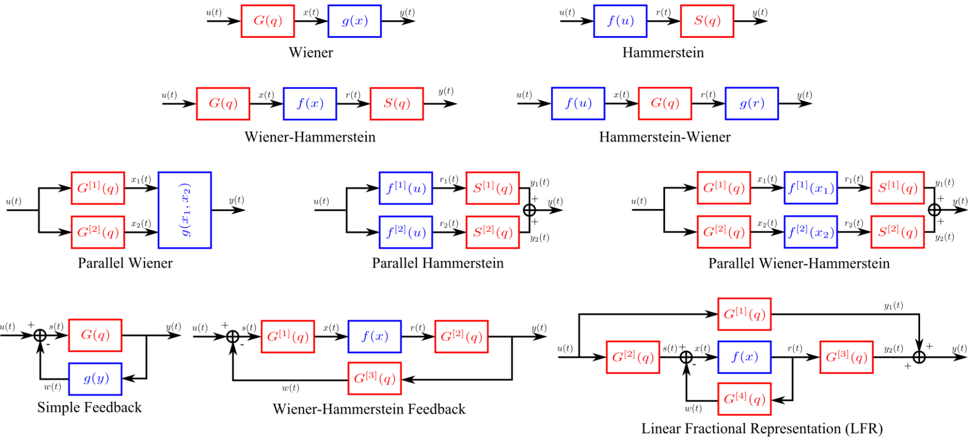


- Wiener
- Hammerstein
- Wiener-Hammerstein
- parallel Wiener-Hammerstein
- nonlinear feedback
- ...

Which model to choose?

Outline

- Which block structure to choose?
- How to identify the chosen block structure?


u(t)
G(q)
x(t)
g(x)
y(t)
Wiener
u(t)
f(u)
r(t)
S(q)
y(t)
Hammerstein
u(t)
G(q)
x(t)
f(x)
r(t)
S(q)
y(t)
Wiener-Hammerstein
u(t)
f(u)
x(t)
G(q)
r(t)
g(r)
y(t)
Hammerstein-Wiener
u(t)
$G^{[1]}(q)$
$x_1(t)$
$G^{[2]}(q)$
$x_2(t)$
$g(x_1, x_2)$
y(t)
Parallel Wiener
u(t)
$f^{[1]}(u)$
$r_1(t)$
$S^{[1]}(q)$
$y_1(t)$
$f^{[2]}(u)$
$r_2(t)$
$S^{[2]}(q)$
$y_2(t)$
+
+
y(t)
Parallel Hammerstein
u(t)
$G^{[1]}(q)$
$x_1(t)$
$f^{[1]}(x_1)$
$r_1(t)$
$S^{[1]}(q)$
$y_1(t)$
$G^{[2]}(q)$
$x_2(t)$
$f^{[2]}(x_2)$
$r_2(t)$
$S^{[2]}(q)$
$y_2(t)$
+
+
y(t)
Parallel Wiener-Hammerstein
u(t)
+
-
s(t)
G(q)
y(t)
g(y)
w(t)
Simple Feedback
u(t)
+
-
s(t)
$G^{[1]}(q)$
x(t)
f(x)
r(t)
$G^{[2]}(q)$
y(t)
$G^{[3]}(q)$
w(t)
Wiener-Hammerstein Feedback
u(t)
$G^{[1]}(q)$
$y_1(t)$
$G^{[2]}(q)$
s(t)
+
-
x(t)
f(x)
r(t)
$G^{[3]}(q)$
$y_2(t)$
+
+
y(t)
$G^{[4]}(q)$
w(t)
Linear Fractional Representation (LFR)

Structure detection

- Bussgang's theorem
- ε – approximation
- Structure detection

Bussgang's Theorem

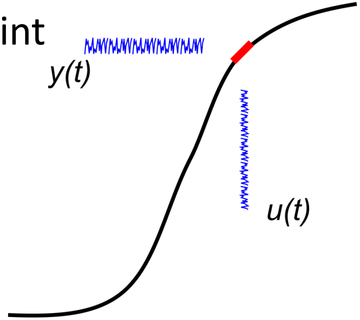
Stationary Gaussian input

→ Static nonlinearity ≈ static gain

$$f(u) = \gamma u$$

ε - Approximation

Small signal around a setpoint



Taylor approximation

→Static nonlinearity ≈ static gain

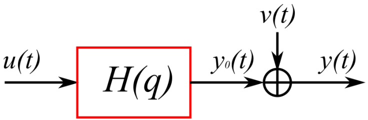
$$f(u) = \gamma u$$

Structure detection

- BLA, ε - Approximation @ different setpoints
  - Change offset
  - Change power spectrum

Structure detection

- Linear-Time-Invariant (LTI)



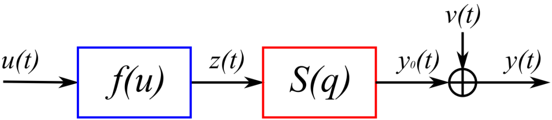
$$G_{bla}(q) = H(q)$$

➔ No changes

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein			
Wiener			
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

- Hammerstein



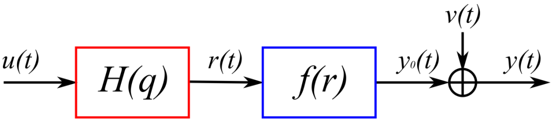
$$G_{bla}(q) = \gamma S(q)$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener			
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

- Wiener



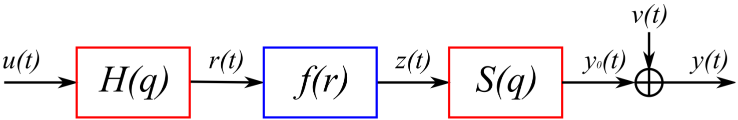
$$G_{bla}(q) = \gamma H(q)$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

- Wiener-Hammerstein



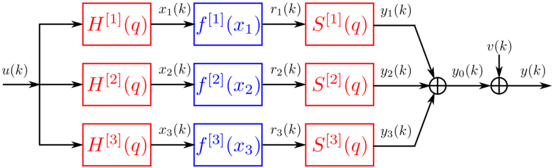
$$G_{bla}(q) = \gamma H(q)S(q)$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH			
Feedback			
LFR			

Structure detection

- Parallel Wiener-Hammerstein



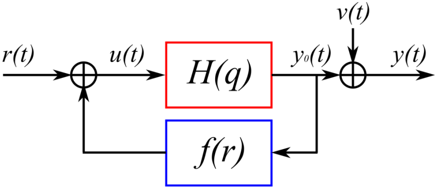
$$G_{bla}(q) = \sum_i \gamma_i H^{[i]}(q) S^{[i]}(q)$$

➔ Moving zeros, fixed poles

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback			
LFR			

Structure detection

- Feedback system



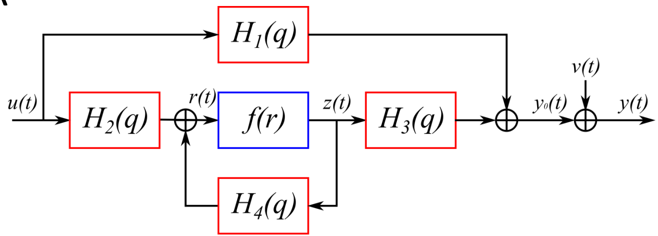
$$G_{\varepsilon}(q) = \frac{H(q)}{1+\gamma H(q)}$$

➔ Fixed zeros, moving poles

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback	Variable	Variable	Fixed
LFR			

Structure detection

- LFR



$$G_\varepsilon(q) = H_1(q) + \frac{\gamma H_2(q) H_3(q)}{1 + \gamma H_4(q)}$$

➔ Moving zeros, moving poles

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback	Variable	Variable	Fixed
LFR	Variable	Variable	Variable

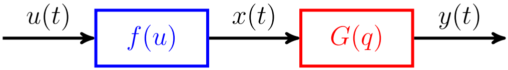
Structure detection

- BLA, ε – approximation @ ≠ setpoints
- Only gain change
  - Hammerstein, Wiener, Wiener-Hammerstein, …
- Zeros shift
  - Parallel feed-forward structure
- Poles shift
  - Feedback present

Outline

- Which block structure to choose?
- How to identify the chosen block structure?
  - Hammerstein model
  - Wiener model
  - Orthonormal basis functions

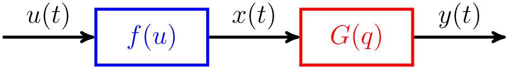
Identification of a Hammerstein model



$$f(u) = \sum_{d=0}^{D} \beta_d u^d$$

$$G(q) = \frac{B(q, \theta)}{A(q, \theta)}$$

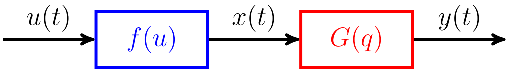
Step 1: Estimate a nonparametric BLA



Random-phase multisine excitation:

$$u^{[m]}(t) = \sum_{k=1}^{N_F} A_k \sin(2\pi k f_0 t + \phi_k^{[m]})$$

Step 1: Estimate a nonparametric BLA



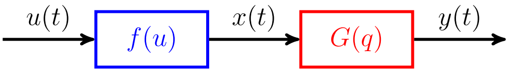
Random-phase multisine excitation:

$$u^{[m]}(t) = \sum_{k=1}^{N_F} A_k \sin(2\pi k f_0 t + \phi_k^{[m]})$$

FRF:

$$G_{BLA}(k) = \frac{1}{M} \sum_{m=1}^{M} \frac{Y^{[m]}(k)}{U^{[m]}(k)}$$

Step 1: Estimate a nonparametric BLA



Random-phase multisine excitation:

$$u^{[m]}(t) = \sum_{k=1}^{N_F} A_k \sin(2\pi k f_0 t + \phi_k^{[m]})$$

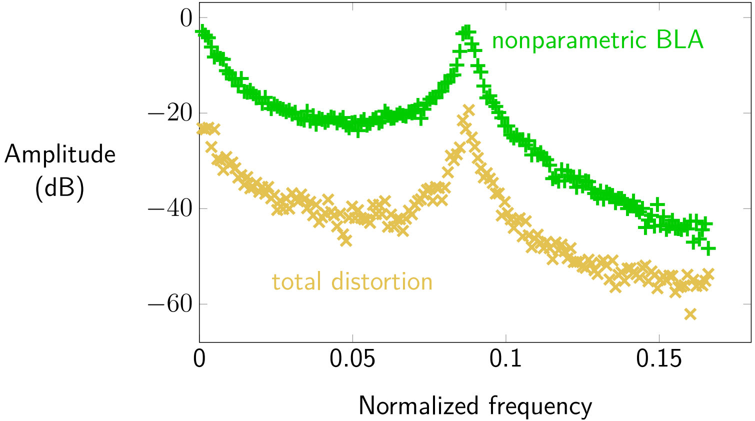
FRF:

$$G_{BLA}(k) = \frac{1}{M} \sum_{m=1}^{M} \frac{Y^{[m]}(k)}{U^{[m]}(k)}$$

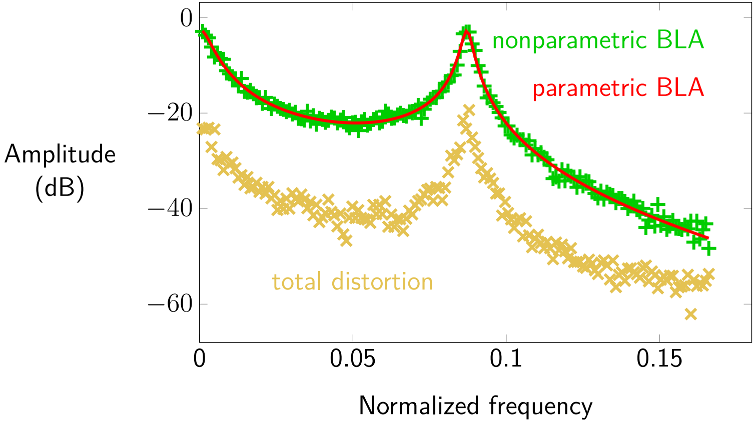
Nonparametric noise model:

$$\sigma^2_{G_{\mathrm{BLA}}}(k) = \sigma^2_{\mathrm{NL}}(k) + \sigma^2_{\mathrm{noise}}(k)$$

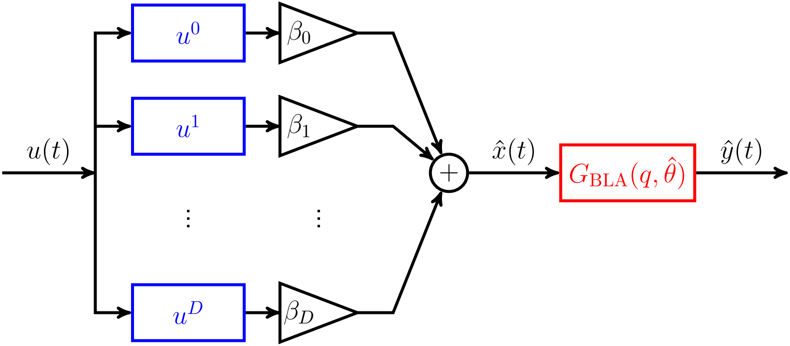
Step 2: Estimate a parametric BLA



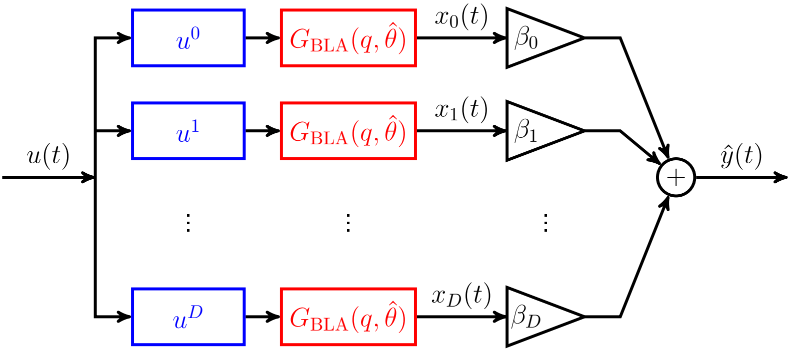
Step 2: Estimate a parametric BLA



Step 3: Estimate the polynomial coefficients

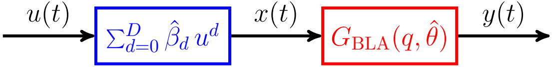


Step 3: Estimate the polynomial coefficients



$$\hat{\beta} = \arg\min_{\beta} \left\| y(t) - \sum_{d=0}^{D} \beta_d x_d(t) \right\|_2^2$$

Step 4: Do a nonlinear optimization of all parameters

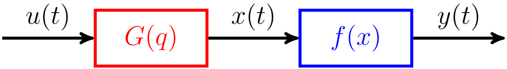


Nonlinear optimization of β and θ simultaneously.

Outline

- Which block structure to choose?
- How to identify the chosen block structure?
  - Hammerstein model
  - Wiener model
  - Orthonormal basis functions

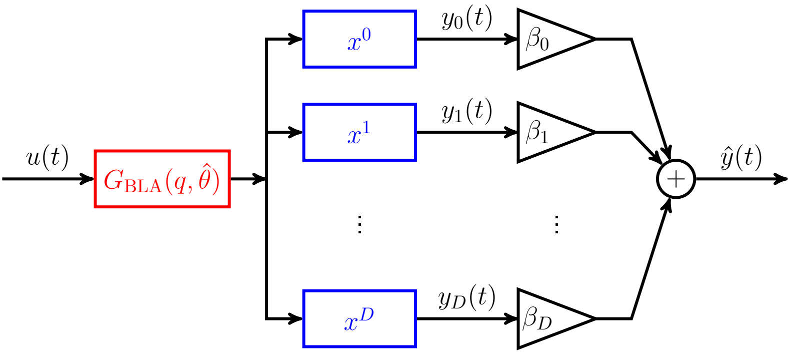
Identification of a Wiener model



$$G(q) = \frac{B(q,\theta)}{A(q,\theta)}$$

$$f(u) = \sum_{d=0}^{D} \beta_d x^d$$

Step 3: Estimate the polynomial coefficients



$$\hat{\beta} = \arg\min_{\beta} \left\| y(t) - \sum_{d=0}^{D} \beta_d y_d(t) \right\|_2^2$$

Outline

- Which block structure to choose?
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  - Hammerstein model
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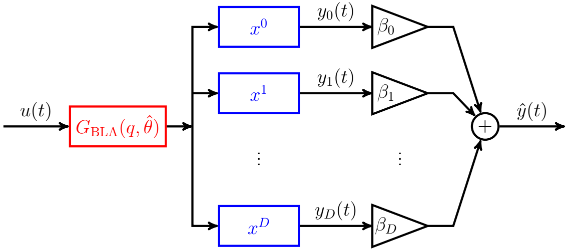
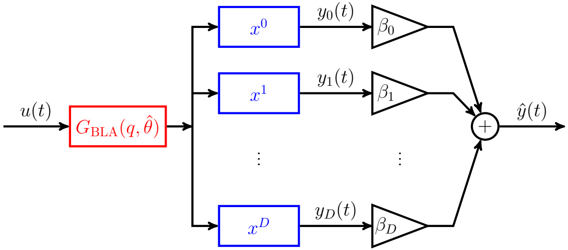
  - Orthonormal basis functions

Motivation for orthonormal basis functions



$$\hat{\beta} = \arg\min_{\beta} \left\| y(t) - \sum_{d=0}^{D} \beta_d y_d(t) \right\|_2^2$$ possibly ill-conditioned

Motivation for orthonormal basis functions



$$\hat{\beta} = \arg\min_{\beta} \left\| y(t) - \sum_{d=0}^{D} \beta_d y_d(t) \right\|_2^2$$

possibly ill-conditioned

Linear dynamics	$\rightarrow$	Rational orthonormal basis functions
Static nonlinearities	$\rightarrow$	Hermite polynomials

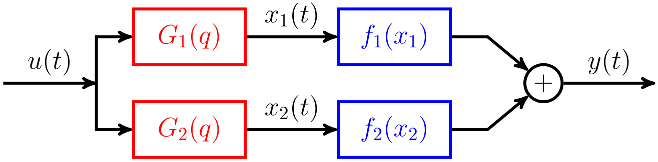
Rational OBFs are determined by their pole locations

$$F_k(q) = \frac{\sqrt{1-|\xi_k|^2}}{q-\xi_k} \prod_{i=1}^{k-1} \frac{1-\xi_i^* q}{q-\xi_i}$$

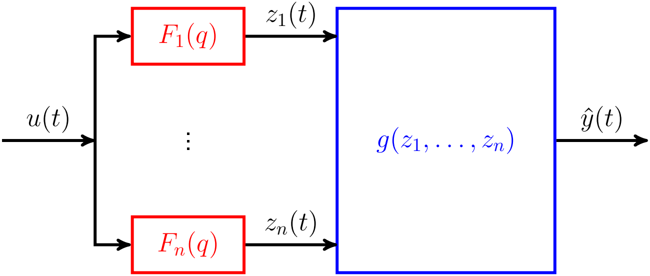
all-pass filter

Pole locations ξ_k	OBFs
origin	FIR
real pole	Laguerre
complex conjugate pair	Kautz
repeated poles	Generalized OBFs
arbitrary	Takenaka-Malmquist

Example: Approximation of a parallel Wiener system

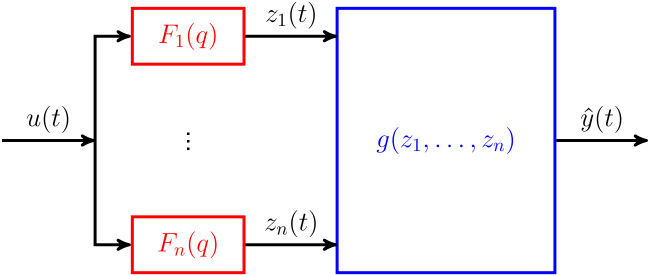


Step 1: Estimate the linear dynamics



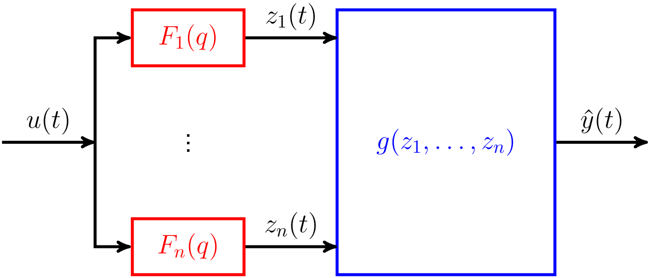
F_k: rational orthonormal basis functions
g: multivariate Hermite polynomials

Step 1: Estimate the linear dynamics



pole locations $\rightarrow$ orthonormal basis functions

Step 1: Estimate the linear dynamics

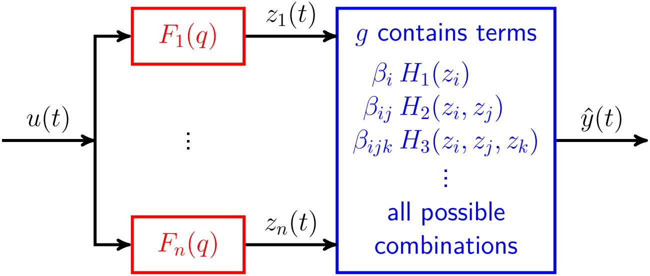


pole locations $\rightarrow$ orthonormal basis functions

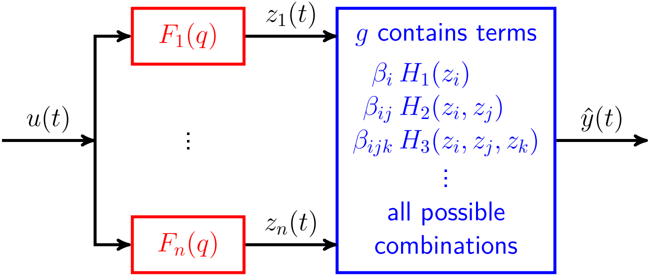
$\Downarrow$

best linear approximation

Step 2: Estimate the polynomial coefficients

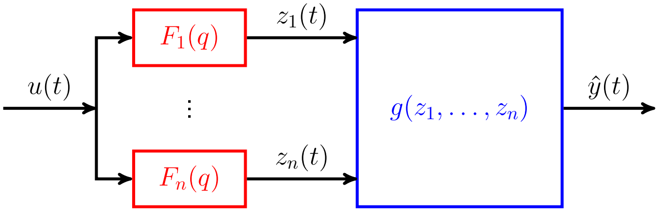


The model is linear-in-the-parameters



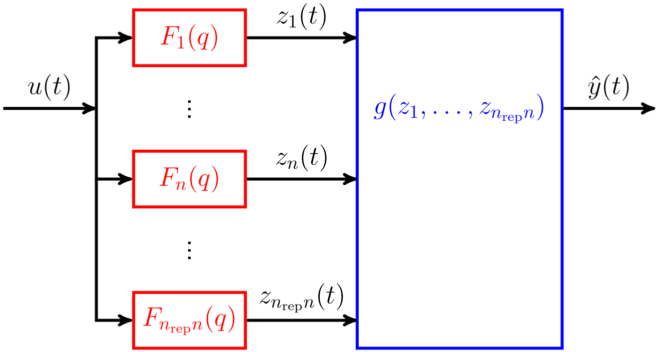
Prior knowledge can be incorporated via user-specified pole locations

$$\{\hat{p}_1, \ldots, \hat{p}_n\} \Rightarrow \{F_1, \ldots, F_n\}$$

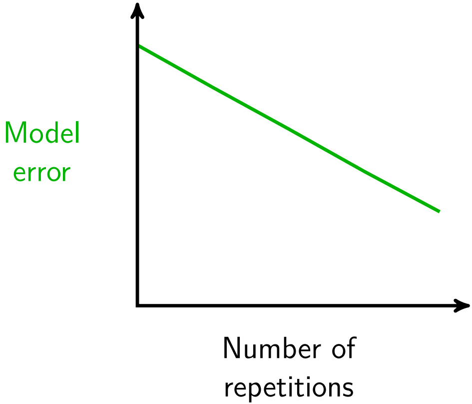


Prior knowledge can be incorporated via user-specified pole locations

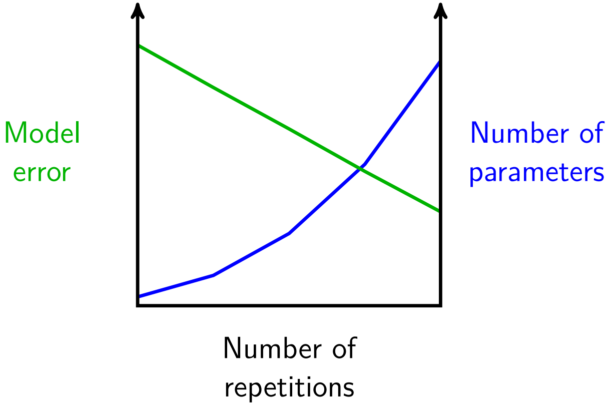
$$
\begin{array}{ccc}
\{\hat{p}_1, \ldots, \hat{p}_n\} & \Rightarrow & \{F_1, \ldots, F_n\} \\
\{\hat{p}_1, \ldots, \hat{p}_n\} & \Rightarrow & \{F_{n+1}, \ldots, F_{2n}\} \\
\vdots & & \vdots
\end{array}
$$



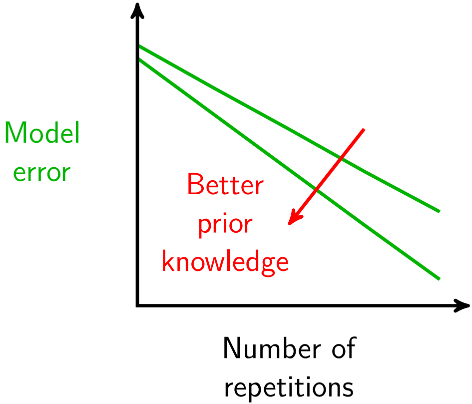
The extra basis functions compensate for a pole mismatch



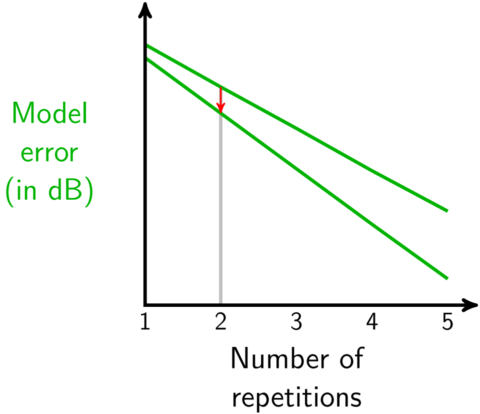
The number of parameters increases rapidly with the number of orthonormal basis functions



Better prior knowledge allows for better models



Iteratively update the pole locations



1) Estimate a high-order nonlinear model
2) Extract a low-order linear model

Overview

- Structure detection via BLA
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener

Vrije Universiteit Brussel

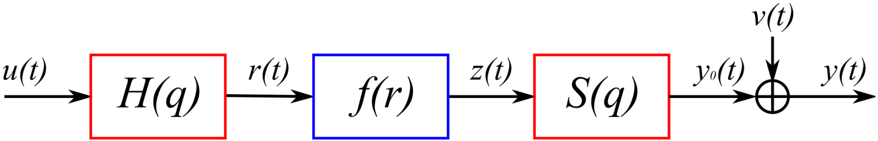
Block-oriented modeling

Koen Tiels, Maarten Schoukens

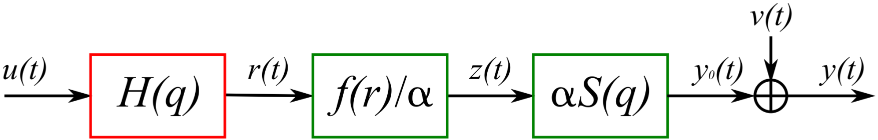
Overview

- Structure detection via BLA / ε-approximation
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener
  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
  - Nonlinear feedback

Wiener-Hammerstein

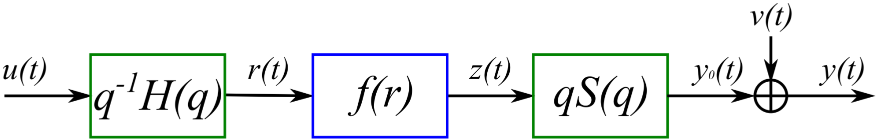


Identifiability



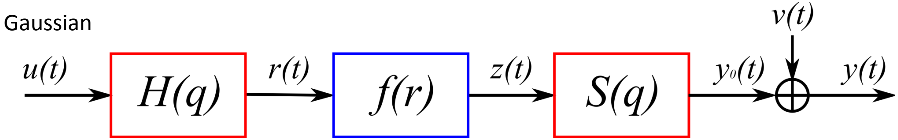
- Gain exchange

Identifiability



- Gain exchange
- Delay exchange

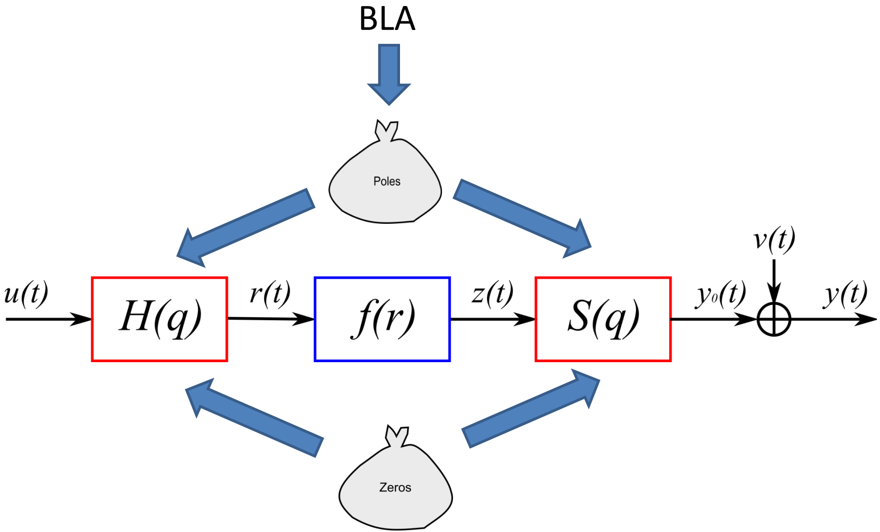
Best Linear Approximation



$$G_{bla}(q) = \gamma H(q) S(q)$$

➔ poles, zeros BLA = poles, zeros system

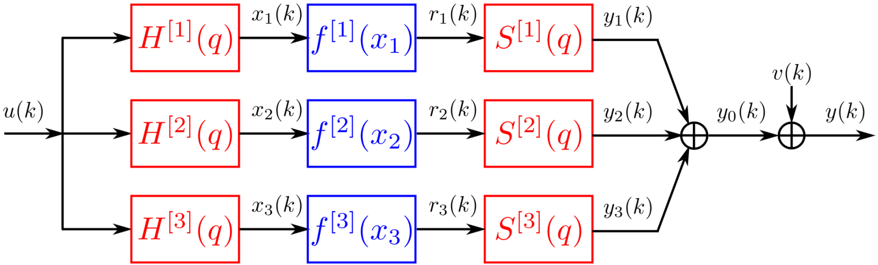
Partition the Dynamics



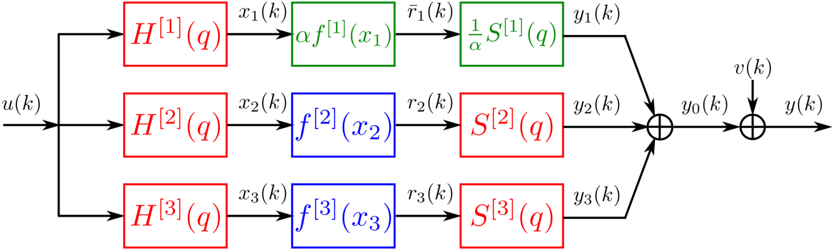
Nonlinear optimization

- Initial parameter values
  - → Optimization of all parameters together
  - → Levenberg-Marquardt algorithm

Parallel Wiener-Hammerstein

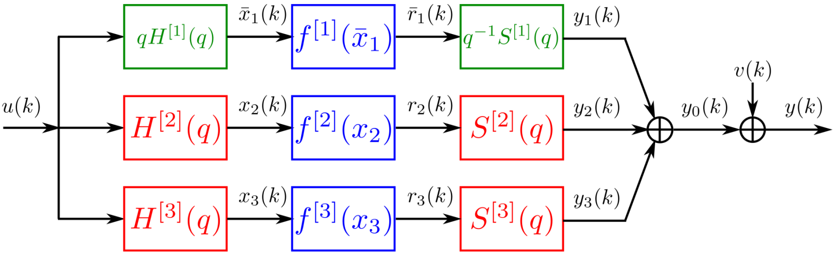


Identifiability



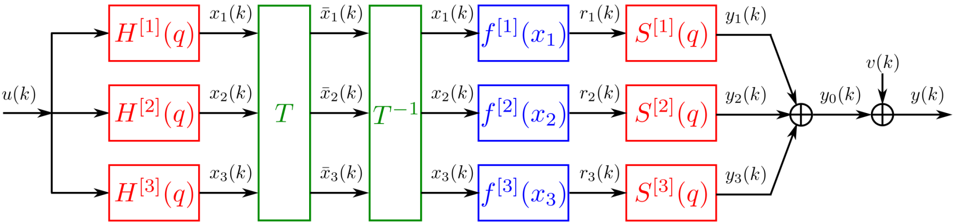
- Gain exchange

Identifiability



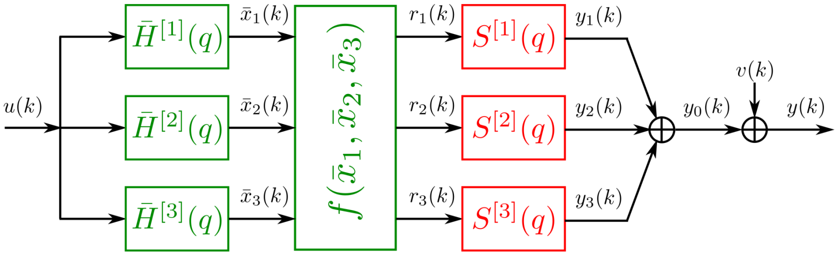
- Gain exchange
- Delay exchange

Identifiability



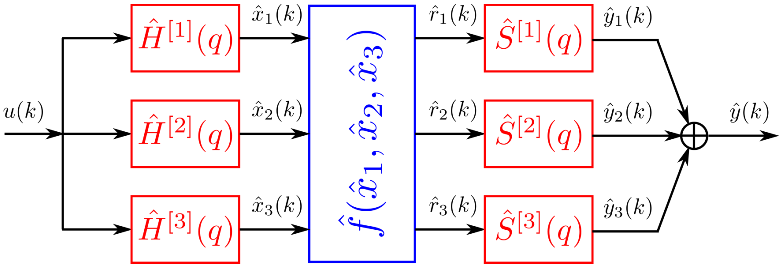
- Gain exchange
- Delay exchange
- Full rank linear transform

Identifiability

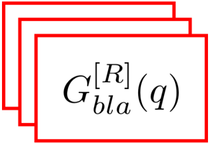


- Gain exchange
- Delay exchange
- Full rank linear transform

Model structure

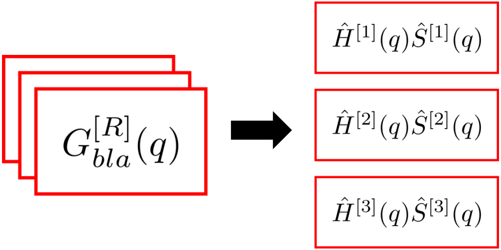


Identification approach



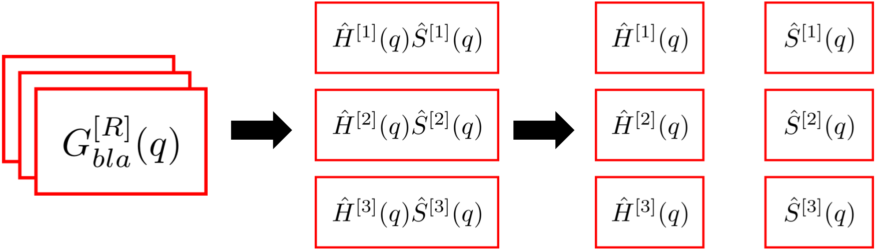
- Estimate overall dynamics

Identification approach



- Estimate overall dynamics
- Decompose the dynamics over the parallel branches

Identification approach

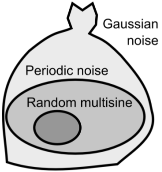


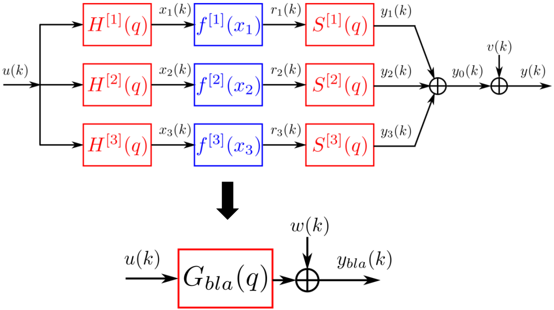
- Estimate overall dynamics
- Decompose the dynamics over the parallel branches
- Partition the dynamics to the front and back

Identification approach

- Identifying the overall dynamics
  - ➔ Best Linear Approximation (BLA)
- Decomposing the dynamics
  - ➔ Singular Value Decomposition (SVD) of the BLAs
- Partition the dynamics
  - ➔ Pole and zero allocation scan

Best Linear Approximation

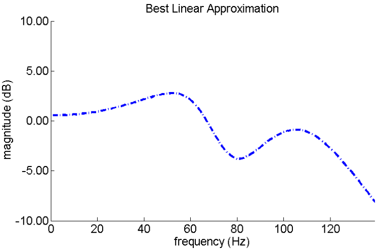


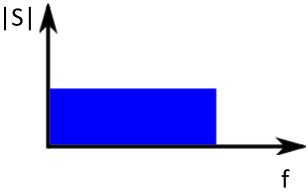


Combination of dynamics!

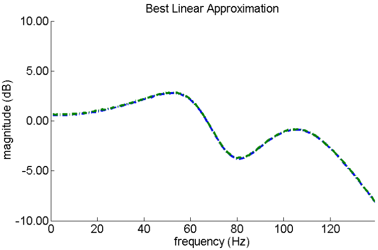
$$G_{bla}(j\omega) = \sum_i \gamma_i H^{[i]}(j\omega) S^{[i]}(j\omega)$$

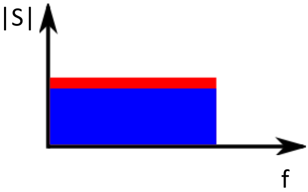
Best Linear Approximation



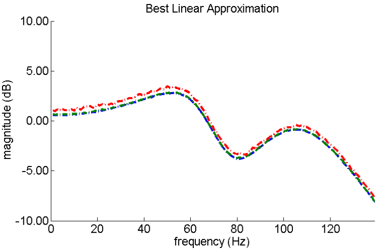


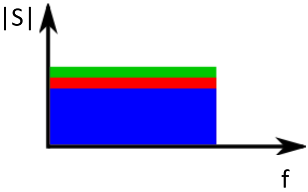
Best Linear Approximation



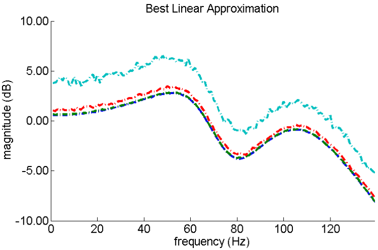


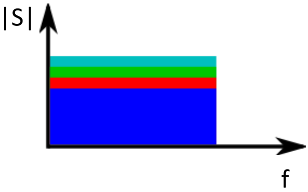
Best Linear Approximation



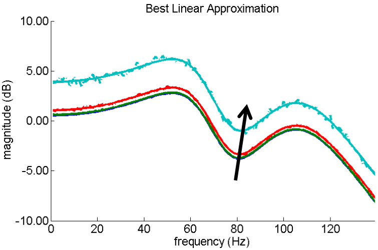


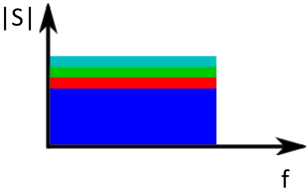
Best Linear Approximation





Best Linear Approximation



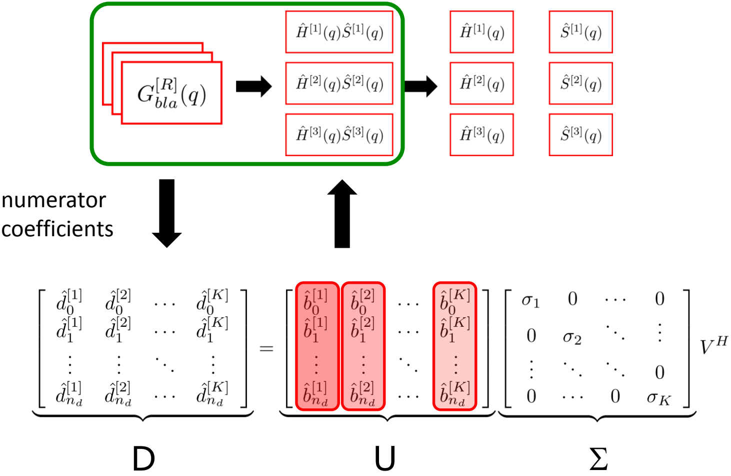


$$G_{bla}(j\omega) = \sum_i \gamma_i H^{[i]}(j\omega) S^{[i]}(j\omega)$$

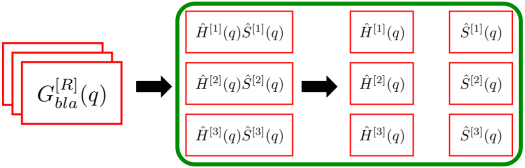
$$\hat{G}_{bla}^{[i]} = \frac{\hat{d}_0^{[i]} + \hat{d}_1^{[i]} q^{-1} + \ldots + \hat{d}_{n_d}^{[i]} q^{-n_d}}{\hat{c}_0 + \hat{c}_1 q^{-1} + \ldots + \hat{c}_{n_c} q^{-n_c}}$$

- Common denominator
  - Fixed poles
  - Moving zeros

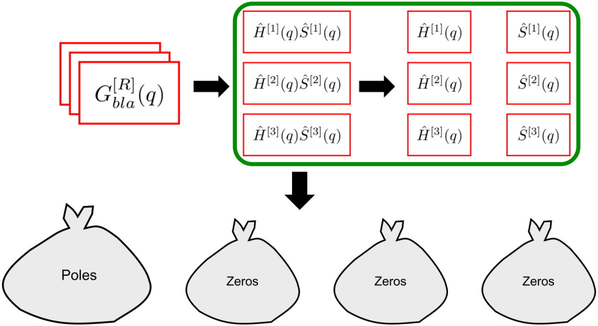
Decomposing the dynamics



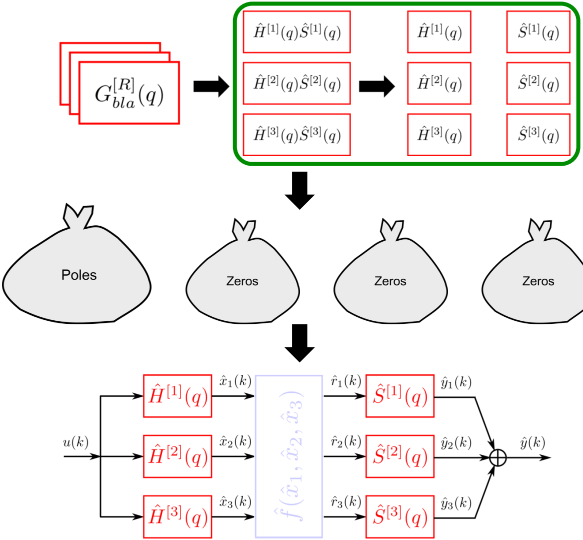
Partition the dynamics



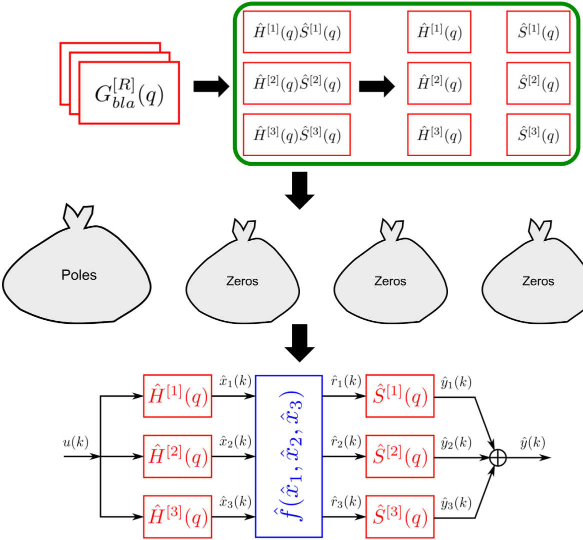
Partition the dynamics



Partition the dynamics



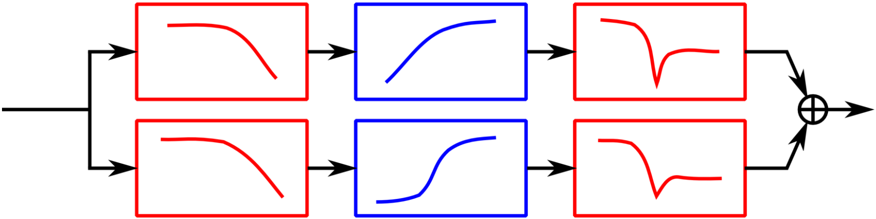
Partition the dynamics



Nonlinear optimization

- Initial parameter values
  → Optimization of all parameters together
  → Levenberg-Marquardt algorithm

Example: test system

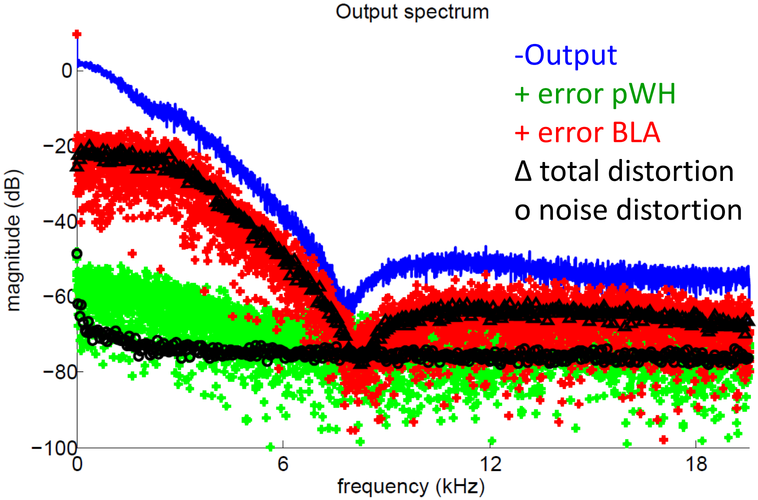


Multisine input:
5 amplitudes
20 realizations
2 periods
16384 samples

System:
Custom built circuit
12^{th} order dynamics
Diode-resistor NL

Model:
2 branches
10 neurons nn NL

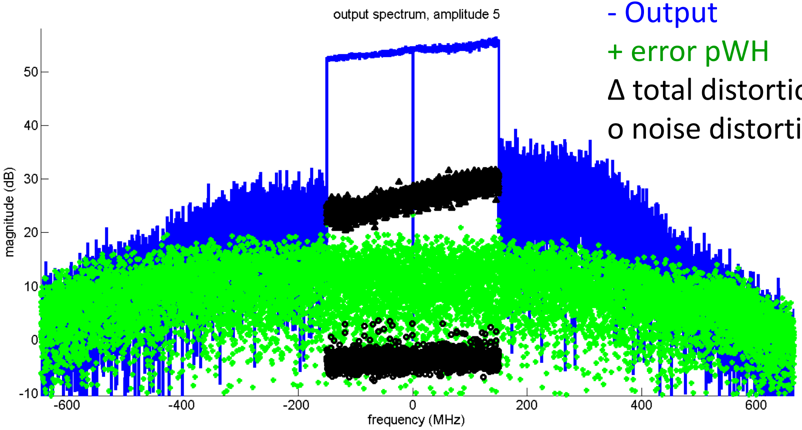
Example: test system



Example: Doherty PA

- Doherty PA
- Input:
  - Multitone, 5 amplitudes, 20 realizations
  - Bandwidth: 300MHz @ 3.45GHz
- Model
  - 2 branches
  - 10 tap FIR BLA
  - 7^{th} order NL

Example: Doherty PA



- Output

+ error pWH

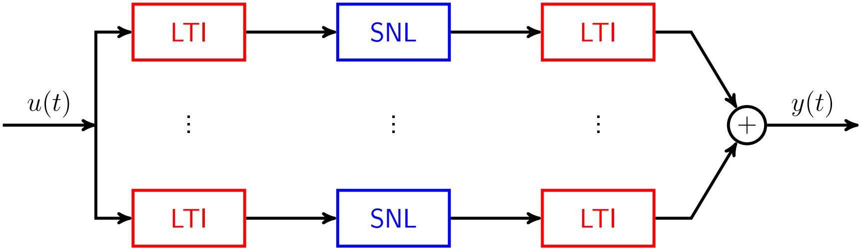
Δ total distortion

o noise distortion

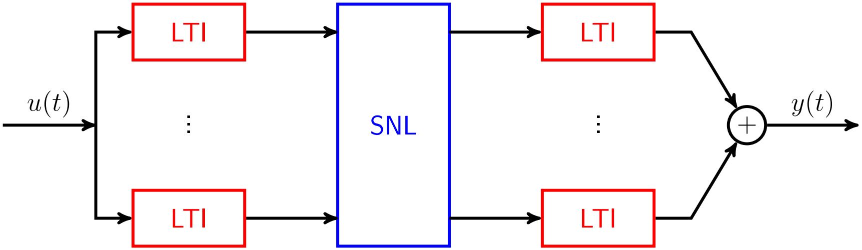
Outline

- Structure detection via BLA / ε-approximation
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener
  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
  - Nonlinear feedback

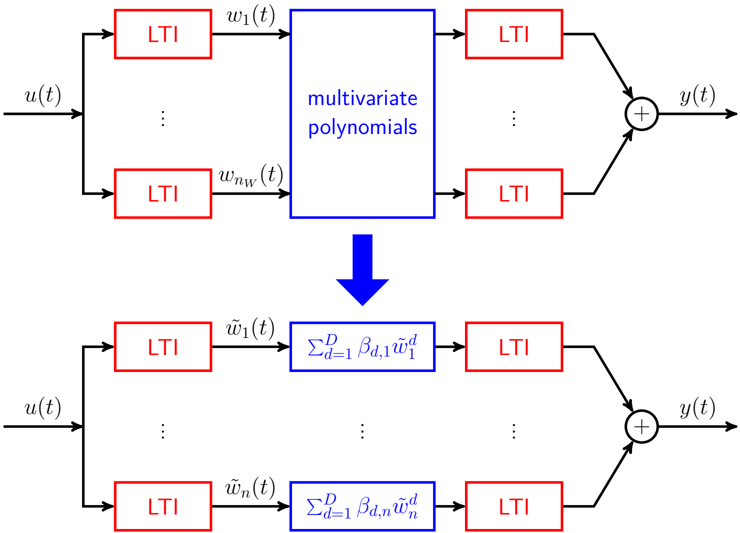
Parallel Wiener-Hammerstein models have good approximation properties



Identifiability issues require a MIMO static nonlinearity



Goal: Eliminate the cross-terms



Homogeneous polynomials are bijectively related to symmetric tensors

- quadratic polynomials $\leftrightarrow$ symmetric matrices

$$a_{11} w_1^2 + 2a_{12} w_1 w_2 + a_{22} w_2^2$$

Homogeneous polynomials are bijectively related to symmetric tensors

- quadratic polynomials $\leftrightarrow$ symmetric matrices

$$
\begin{aligned}
a_{11}w_1^2 + 2a_{12}w_1w_2 + a_{22}w_2^2 &= \mathbf{w}^T\mathbf{A}\mathbf{w} \\
&= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\
&= \mathbf{A} \ \bar{\times}_1 \ \mathbf{w} \ \bar{\times}_2 \ \mathbf{w}
\end{aligned}
$$

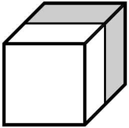
Homogeneous polynomials are bijectively related to symmetric tensors

- quadratic polynomials $\leftrightarrow$ symmetric matrices

$$
\begin{aligned}
a_{11} w_1^2 + 2a_{12} w_1 w_2 + a_{22} w_2^2 &= \mathbf{w}^T \mathbf{A} \mathbf{w} \\
&= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\
&= \mathbf{A} \bar{\times}_1 \mathbf{w} \bar{\times}_2 \mathbf{w}
\end{aligned}
$$

- cubic polynomials $\leftrightarrow$ symmetric third-order tensors

$$
\begin{aligned}
x_{111} w_1^3 + 3x_{112} w_1^2 w_2 + 3x_{122} w_1 w_2^2 + x_{222} w_2^3 \\
= \boldsymbol{\mathcal{X}} \bar{\times}_1 \mathbf{w} \bar{\times}_2 \mathbf{w} \bar{\times}_3 \mathbf{w}
\end{aligned}
$$

$\boldsymbol{\mathcal{X}} =$  $\rightarrow \left[\begin{array}{cc|cc} x_{111} & x_{112} & x_{112} & x_{122} \\ x_{112} & x_{122} & x_{122} & x_{222} \end{array}\right]$

Homogeneous polynomials are bijectively related to symmetric tensors

- quadratic polynomials $\leftrightarrow$ symmetric matrices

$$a_{11} w_1^2 + \cancel{2 a_{12} w_1 w_2} + a_{22} w_2^2 = \mathbf{w}^T \mathbf{A} \mathbf{w}$$

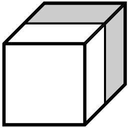
$$= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} a_{11} & \cancel{a_{12}} \\ \cancel{a_{12}} & a_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$= \mathbf{A} \bar{\times}_1 \mathbf{w} \bar{\times}_2 \mathbf{w}$$

- cubic polynomials $\leftrightarrow$ symmetric third-order tensors

$$x_{111} w_1^3 + \cancel{3 x_{112} w_1^2 w_2} + \cancel{3 x_{122} w_1 w_2^2} + x_{222} w_2^3$$

$$= \boldsymbol{\mathcal{X}} \bar{\times}_1 \mathbf{w} \bar{\times}_2 \mathbf{w} \bar{\times}_3 \mathbf{w}$$

$\boldsymbol{\mathcal{X}} =$  $\rightarrow$

$$\left[\begin{array}{cc|cc} x_{111} & \cancel{x_{112}} & \cancel{x_{112}} & \cancel{x_{122}} \\ \cancel{x_{112}} & \cancel{x_{122}} & \cancel{x_{122}} & x_{222} \end{array}\right]$$

Decouple quadratic polynomials via an eigenvalue decomposition (EVD)

$$
\begin{aligned}
a_{11} w_1^2 + 2 a_{12} w_1 w_2 + a_{22} w_2^2 &= \mathbf{w}^T \mathbf{A} \mathbf{w} \\
&\overset{EVD}{=} \mathbf{w}^T \left(\mathbf{V} \mathbf{D} \mathbf{V}^T \right) \mathbf{w} \\
&= \left(\mathbf{w}^T \mathbf{V} \right) \mathbf{D} \left(\mathbf{V}^T \mathbf{w} \right) \\
&= \tilde{\mathbf{w}}^T \mathbf{D} \tilde{\mathbf{w}} \\
&= \sum_{r=1}^{2} d_r \left(v_{1r} w_1 + v_{2r} w_2 \right)^2 \\
&= d_1 \tilde{w}_1^2 + d_2 \tilde{w}_2^2
\end{aligned}
$$

A similar decomposition for tensors?

- Eigenvalue decomposition (EVD):

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}$$

$$= d_1 \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \begin{bmatrix} v_{11} & v_{21} \end{bmatrix} + d_2 \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} \begin{bmatrix} v_{12} & v_{22} \end{bmatrix}$$

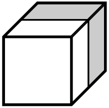
A similar decomposition for tensors: the CPD

- Eigenvalue decomposition (EVD):

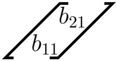
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} = d_1 \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \begin{bmatrix} v_{11} & v_{21} \end{bmatrix} + d_2 \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} \begin{bmatrix} v_{12} & v_{22} \end{bmatrix}$$

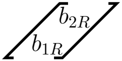
- (Symmetric) canonical polyadic decomposition (CPD):

$$\boldsymbol{\mathcal{X}} =$$



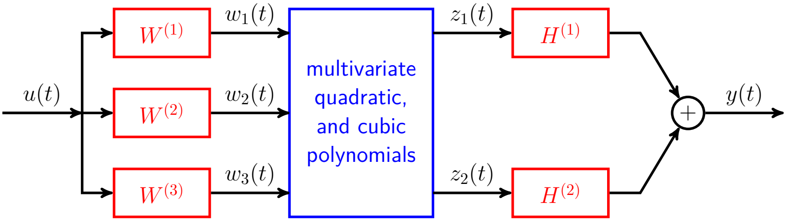
$$\rightarrow \left[\begin{array}{cc|cc} x_{111} & x_{112} & x_{112} & x_{122} \\ x_{112} & x_{122} & x_{122} & x_{222} \end{array}\right]$$





$$\approx \lambda_1 \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \begin{bmatrix} b_{11} & b_{21} \end{bmatrix} + \ldots + \lambda_R \begin{bmatrix} b_{1R} \\ b_{2R} \end{bmatrix} \begin{bmatrix} b_{1R} & b_{2R} \end{bmatrix}$$

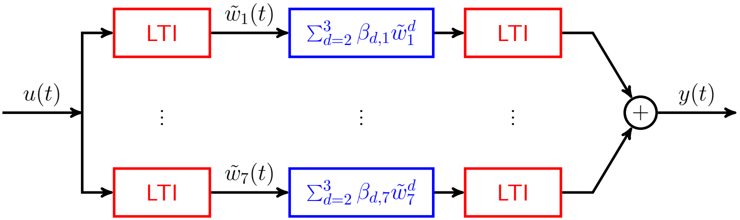
Example



$$a^{(k)}_{i_1 i_2} \text{ and } x^{(k)}_{i_1 i_2 i_3} \in \mathcal{N}(0,1)$$

There are 32 polynomial coefficients.

Example

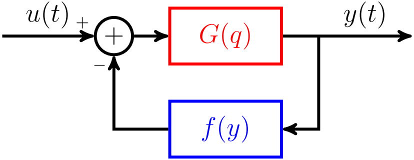


There are only 14 polynomial coefficients.

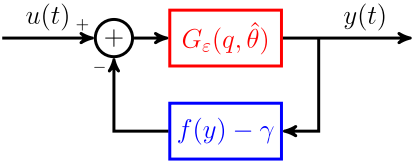
Outline

- Structure detection via BLA / ε-approximation
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener
  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
  - Nonlinear feedback

Identification of a nonlinear feedback model

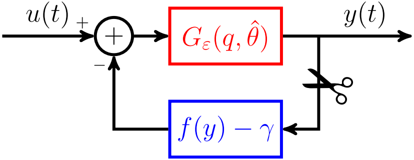


Step 1: Estimate the linear dynamics

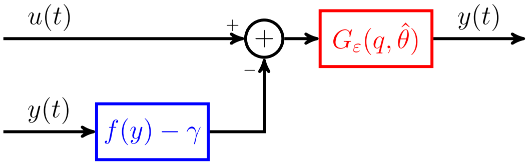


$$G_\varepsilon(q,\hat{\theta}) = \frac{B_\varepsilon(q,\hat{\theta})}{A_\varepsilon(q,\hat{\theta})}$$

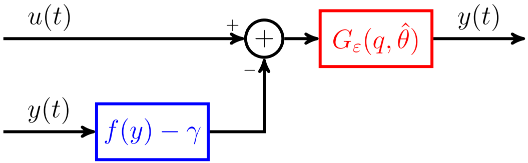
Step 2: Estimate the static nonlinearity



Step 2: Estimate the static nonlinearity



Step 2: Estimate the static nonlinearity

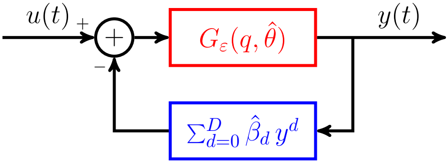


$$w(t) = y(t) - G_\varepsilon(q,\hat{\theta})u(t)$$

$$\hat{w}(t) = -G_\varepsilon(q,\hat{\theta})\left(\sum_{d=0}^{D}\beta_d y^d(t)\right)$$

$$\hat{\beta} = \arg\min_{\beta} \|w(t) - \hat{w}(t)\|_2^2$$

Step 3: Optimize all model parameters



Nonlinear optimization of β and θ simultaneously.

Overview

- Structure detection via BLA / ε-approximation
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener
  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
  - Nonlinear feedback

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- Guidelines

Model structure selection

Do I need a nonlinear model?

Best linear approximation framework:

Noise distortions $\leftrightarrow$ Nonlinear distortions

Which block-oriented model is suited?

Best linear approximation at different setpoints:

- Shifting poles $\Rightarrow$ Nonlinear feedback
- Shifting zeros $\Rightarrow$ Parallel nonlinear signal paths

Input design

What inputs are well suited?

- Model errors $\Rightarrow$ use realistic excitations (amplitude/frequency)
- BLA framework $\Rightarrow$ preferably periodic random signals
  $\sigma^2_{noise} \sim O\left(\frac{1}{\#\text{periods } \#\text{realizations}}\right)$
  $\sigma^2_{nonlinear} \sim O\left(\frac{1}{\#\text{realizations}}\right)$
- Multisine excitation/perturbation signals
  $\Rightarrow$ full control of amplitude spectrum