

Line fitting

problem: fit points $d_1, \dots, d_N \in \mathbb{R}^2$ by a line

1. find condition for existence of a line (any line in \mathbb{R}^2) that passes through the points
2. how would you test the condition in MATLAB?
3. implement a method for exact line fitting

Conic section fitting

problem: fit points $d_1, \dots, d_N \in \mathbb{R}^2$ by conic section

$$\mathcal{B}(S, u, v) = \{ d \in \mathbb{R}^2 \mid d^\top S d + u^\top d + v = 0 \}$$

1. find condition for existence of an exact fit
2. propose numerical method for exact fitting
3. implement the method and test it on the data

$$d_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad d_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad d_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Recursive sequence fitting

problem: fit $w = (w(1), \dots, w(T))$ by model

$$\mathcal{B} = \{ w \mid R_0 w + R_1 \sigma w + \dots + R_\ell \sigma^\ell w = 0 \}$$

1. find condition for existence of an exact fit
first, with, and then, without knowledge of ℓ
2. propose numerical method for exact fitting
find the smallest ℓ , for which exact model exists
3. implement the method and test it on the data

(1, 2, 4, 7, 13, 24, 44, 81)