Noise filtering demo

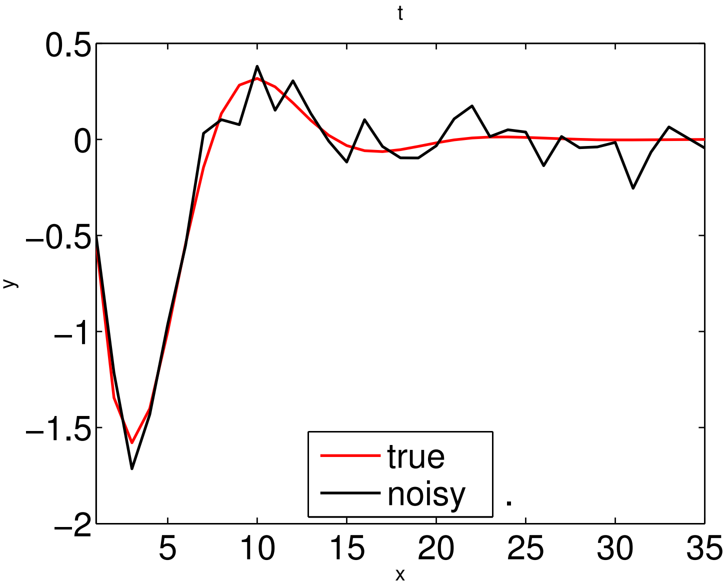
Ivan Markovsky

Data generation

```
n = 2; sys0 = drss(n, 1, 0); load sys0
y0 = initial(sys0, ones(n, 1));
s = 0.2; T = size(y0, 1); yn = randn(T, 1);
y = y0 + s * yn * norm(y0) / norm(yn);
```

- `drss(n, p, m)` `n`-th order discrete-time random LTI state space model with `p` output and `m` inputs
- `initial(sys, xini)` free response of a state-space LTI system `sys` to initial condition `xini`
- `randn(m, n)` $\mathtt{m} \times \mathtt{n}$ zero mean random matrix with uncorrelated normally distributed elements

True and noisy signals



Smoothing (low-pass filter) data

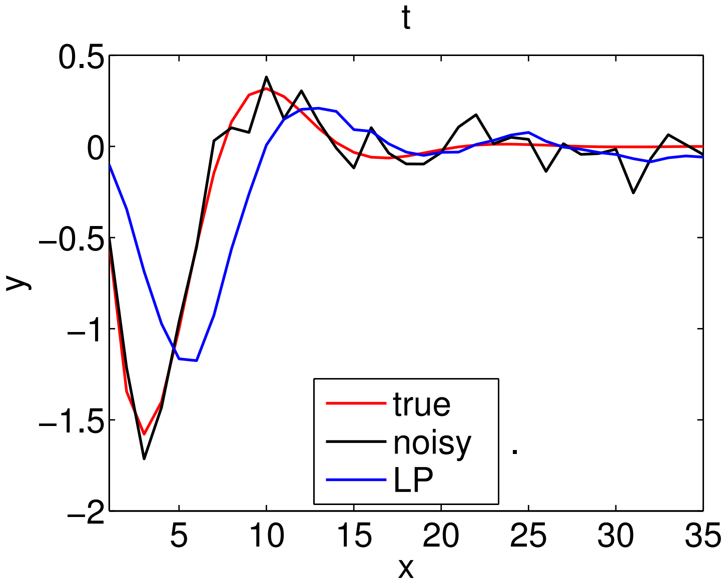
```
N = 5; b = 1 / N * ones(1, N);
yh1 = filter(b, 1, y);
```

- `filter(b, a, u)` filters the signal `u` by the filter

$$
\begin{aligned}
a_0 y(t) + a_1 y(t-1) + \cdots + a_n y(t-n) \\
= b_0 u(t) + b_1 u(t-1) + \cdots + b_m u(t-m)
\end{aligned}
$$

- `N = 5; b = 1 / N * ones(1, N);`
  defines a moving average (MA) filter with 5 taps

Low-pass filtered signal



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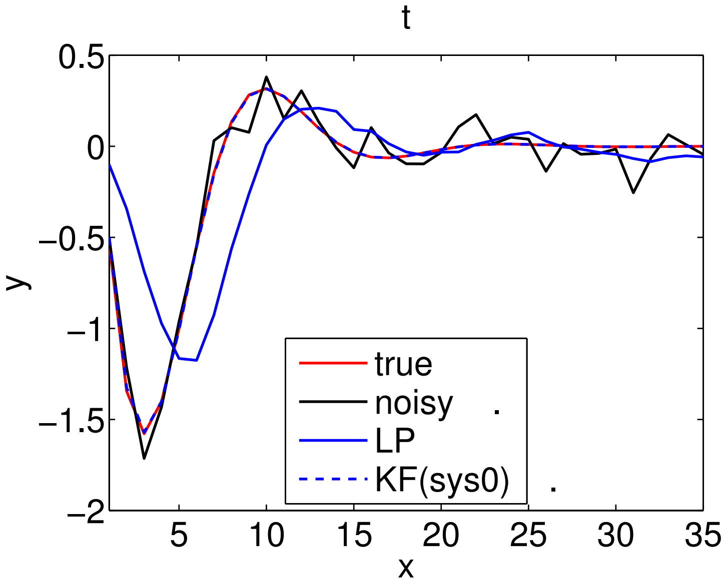
Kalman filter with the true model

```
z0 = eig(sys0)';
V = z0(ones(T, 1), :) .^ t(:, ones(1, n));
yh2 = V * (V \ y);
```

- `eig(sys)` computes the poles of `sys`
- `V` is the Vandermonde matrix $V = \begin{bmatrix} z_1^0 & \cdots & z_n^0 \\ z_1^1 & \cdots & z_n^1 \\ \vdots & & \vdots \\ z_1^T & \cdots & z_n^T \end{bmatrix}$
- `x = A\b` is the least-squares approximate solution of an overdetermined system of linear equations $Ax = b$

Optimally filtered signal with the true model



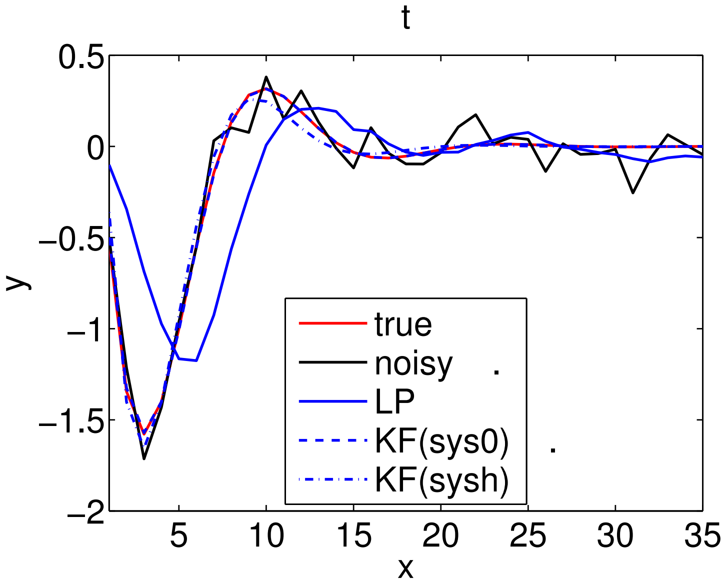
Kalman filter with estimated model

```
zh = eig(h2ss(y, n))';
V = zh(ones(T, 1), :) .^ t(:, ones(1, n));
yh3 = V * (V \ y);
```

- `h2ss` is an state space LTI model of order `n` with impulse response ≈ `y`
- `h2ss` implements a subspace identification method, see Section 3.1 of

  http://homepages.vub.ac.be/~imarkovs/book.html

Optimally filtered signal with estimated model



Estimation errors

```
[norm(y0 - yh1) norm(y0 - yh2) norm(y0 - yh3)]
```

- `norm(x)` computes the 2-norm of the vector `x`
- the obtained results are as follows

	MA-5 filter	KF(sys0)	KF(h2ss(y))
`norm(y0-yh)`	2.0526	0.042383	0.31086