Mini-project: Dynamic measurement

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The task of this mini-project is to improve the speed and accuracy characteristics of a sensor by real-time signal processing. It turns out that this is an application of Kalman filtering. The classical approach is identification of a model of the sensor dynamics followed by model-based filter design and implement on a DSP. The project has three main tasks: 1) mathematical formalization of the problem, 2) development of solution methods, and 3) implementation and testing of the methods. The testing is done on an inexpensive laboratory setup, using the Lego Mindstorms educational kit in combination with a temperature sensor.

1 Introduction

The first task of any problem is to understanding the problem. In the problem at hand, this is to understand the high-level statement "improve the speed and accuracy characteristics of sensor by real-time signal processing". Then, the task is to formalize the high-level problem by formulating a well defined mathematical problem. Finally, the task is to develop an algorithm for solving the mathematical problem. Finally, the solution should be implemented and tested.

1.1 Understanding the problem

As a motivating exercise, consider the following problem.

> "A thermometer reading 21°C, which has been inside a house for a long time, is taken outside. After one minute the thermometer reads 15°C; after two minutes it reads 11°C. What is the outside temperature? (According to Newton's law of cooling, an object of higher temperature than its environment cools at a rate that is proportional to the difference in temperature.)"

This problem is a simplified version of the project (first order scalar measurement process, exact data, three samples), which still shows the main idea—a "slow" processes can be made faster by data processing. The project is a generalization of the problem and its solution to higher order multivariable processes, noisy data, and real-time data acquisition.

The first task is to understand the problem.

- What is the given data and what is to be found?
- How are the data and the to-be-found variables related?

It is important to separate the answers of these questions from the subsequent search for solution methods.

In practice, the sensor reading is sampled in time, quantized, and collected in real-time. Initially, abstract from the complications resulting from the quantization and the real-time data collection. However, you will address these issues later. In particular, the real-time signal processing is an essential aspect in implementation of the method in practice.

The to-be-found parameter is the measured value. In metrology, it is natural to assume that it is constant over the measurement time. You can relax this assumption, *i.e.*, consider tracking of a time-varying quantity, in a follow-up project. Since the measurement process starts at an initial moment of time, there is a step change in the measured variable. This step change initiates a transient. The transient is the key object of interest in the dynamic measurement problem.

The connection between the data and the measured variable is the sensor dynamics. The sensor is based on a physical process, *e.g.*, heat exchange between the environment and the thermometer in the case of temperature measurement. This is a link between engineering and physics, which leads to modeling using the underlying physical laws.

After the problem is conceptually understood, the task is to formalize it mathematically.

1.2 Mathematical formalization

The link between the problem at hand—dynamic measurement—and system theory is done by modeling the sensor as a dynamical system, with input u the measured variable and output y the sensor's reading (see Figure 1). The measured variable is assumed constant during the measurement period, *i.e.*, $u(t) = \bar{u}$, where $\bar{u}$ is the

to-be-measured value u (constant during measurement) —sensor→ measured value y (exhibits transient response)

Figure 1: The measurement process is modeled as an input-output dynamical system, where the input is the to-be-measured variable and the output is the value measured by the sensor.

to-be-estimated measured variable.

1.3 Implementation and testing

The crucial step of the mini-project is deriving an algorithm for the estimation of the parameter of interest $\bar{u}$. This algorithm can be

- model based or data-driven (model-free),
- off-line (batch processing of the data) or on-line (real-time processing of the data),
- exact (assuming the data is noise free) or approximate (can deal with noise in the dat).

For practical implementation data-driven, on-line, approximate algorithm is needed however in developing the solution it is better to proceed from simple (model-based, off-line, exact solution) to the more complicated realistic situation of having no model, processing the data in real-time, and having noisy observations.

2 Sample results

Sample results on data obtained from human temperature measurement and the corresponding optimal fit by the identified model are shown in Figure 2, left. The real-time prediction on the same data, obtained with the Kalman filter designed for an identified model, is shown in the middle plot. Since the prediction of the Kalman filter is tested on the same data that was used for the model identification, the results are not representative for the actual performance of the method. The right plot in Figure 2 verifies the robustness of the Kalman filter by applying it on new data—the temperature of another subject.

3 Generalizations

Three possible generalizations of the dynamic measurement problem described are using data from multiple sensors and estimation of a non-constant measured value. The generalization of the dynamic measurement problem to using data from multiple sensors is known as *data fusion*. This generalization has important practical implications. For example, it shows how to

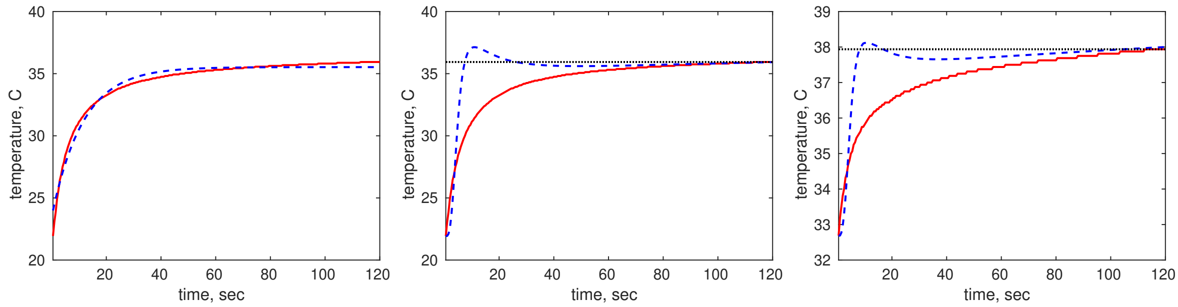

Figure 2: Left: The model's output (dashed line) fits the data (solid line). The data resembles an exponential function plus a trend, *i.e.*, it does not satisfy the assumption of a first order linear time-invariant dynamics. Middle: The prediction of the Kalman filter (dashed line) converges to the measured temperature (dotted line) faster than the natural response of the sensor (solid line). Right: Prediction of the Kalman filter (dashed line) on different data than the one used for identification still converges to the measured temperature (dotted line) faster than the natural response of the sensor (solid line). This demonstrates the robustness of the method to model uncertainty.

- build an accurate sensors using inaccurate sensors,
- fuse measurements that are not synchronized in time, and
- fuse data from sensors with different dynamics.

The improvements obtained by using measurements from multiple sensors is not on the level of the hardware (measurement technique) but on the level of the software that processes the measurements, so that it does not depend on the particular type of sensor.

The generalization of the dynamic measurement method for measuring of non-constant value assumes that the measured value is linearly parameterized, so that the problem is to estimate the parameters of the basis expansion. When the basis functions are complex exponentials, the problem is again solved by a Kalman filter, designed for an augmented system that includes the model of the sensor and the input model.

Consider measuring temperature by two thermometers. The basic question considered is how to combine the two measurements into one that is better than either of the individual measurements. The simple idea of averaging point-wise in time the measurements of the sensors is problematic for a number of reasons. First, the two thermometers may start measuring at different moments of time, which means that their readings are not synchronized in time. Second, the initial conditions (*i.e.*, thermometers' initial temperatures) may not be the same, which again leads to a synchronization problem. Finally, the time constants of the thermometers may be different, so that even if the initial conditions are the same and the starting moments are the same, the transients have different time-constants. Consider for example measuring temperature by a thermometer that is slow but accurate and another thermometer that is fast but inaccurate. The questions of how to combine the two measurements in an optimal way and how to compute the estimate efficiently in real-time are nontrivial. As another example, consider a closed container with constant volume. By Boyle's law, the pressure and the temperature in the container are related. This means that we can fuse data from a thermometer and a pressure sensor.

4 Learning outcomes

Generic learning outcomes from this mini-project are identifying, critically analyzing, and testing assumptions; formulating and testing hypothesis; performing simulation and real-life experiments. Problem specific learning outcomes are derivation, implementation, and validation of a Kalman filter.