Identification of Dynamical Systems

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Getting to know each other

Where and on what subject you've done bachelor?
Did you have identification courses before?

What is the topic of your master thesis?

What do you expect from this course?

Exercises and mini-projects

"I hear, I forget;
I see, I remember;
I do, I understand." *Chinese philosopher*

analytical and computer exercises in class
(bring your laptop with Matlab)

weekly homework assignments

mini-projects

Evaluation

by doing the homework assignments, you earn 10 points

final mark $= 10 + x$, where x is the mark from the WPO

report and oral presentation of the mini-project

Outline

What is a *dynamical system*?

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input/output map

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set of trajectories (behavior)

How to *represent* a dynamical system?

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parametric

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- differential / difference equation

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- differential / difference equation
- rational transfer function

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- rational transfer function
- state space model

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- differential / difference equation
- rational transfer function
- state space model

non-parametric

How to *represent* a dynamical system?

parametric

- differential / difference equation
- rational transfer function
- state space model

non-parametric

- impulse response

How to *represent* a dynamical system?

parametric

- differential / difference equation
- rational transfer function
- state space model

non-parametric

- impulse response
- frequency response

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data-driven

Algorithms: How to convert one representation into another?

state construction

realization theory

Kung's method

Analysis: checking properties of the system

stability

controllability

observability

Outline

How to check if the data is exact?

with a given model

without a given model

different representations lead to different algorithms

Identifiability: When the data can be recovered back from the data?

persistency of excitation

the fundamental lemma

generalizations

Algorithms for exact identification

subspace methods

N4SID (Bart De Moor, KUL)

MOESP (Michel Verhaegen, Delft)

Outline

What do we mean by "noisy" data?

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measurement noise

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- output error
- errors-in-variables

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model uncertainty

What are we after?

maximum-likelihood estimator

approximation criterion

dichotomy: deterministic vs stochastic

Algorithms for approximate identification

PEM

structured low-rank approximation (SLRA)

regularization methods

References

D. Luenberger, Introduction to dynamical systems: Theory, models and applications

I. Markovsky, Exact and approximate modeling in the behavioral setting

- Chapter 7: Introduction to dynamical models
- Chapter 8: Exact identification
- Chapter 11: Approximate system identification

I. Markovsky. Low-Rank Approximation: Algorithms, Implementation, Applications

J.-W. Polderman and J. Willems, Introduction to mathematical systems theory

Homework

"open-ended" problem statement

given a time-series

$$w_{\mathrm{d}} = \begin{bmatrix} u_{\mathrm{d}} \\ y_{\mathrm{d}} \end{bmatrix} = \left(\begin{bmatrix} u_{\mathrm{d}}(1) \\ y_{\mathrm{d}}(1) \end{bmatrix}, \ldots, \begin{bmatrix} u_{\mathrm{d}}(T) \\ y_{\mathrm{d}}(T) \end{bmatrix} \right)$$

check if it is trajectory of linear time-invariant (LTI) system

your tasks

formalize the statement "w_{d} is trajectory of LTI system"
make it mathematical: what does it mean exactly?

derive an algorithm that implements the test

construct a simulation example to test it