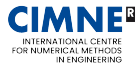


The Behavioral Toolbox

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How to check if two LTI systems are equal?

we would like this MATLAB code to give 'true'

```
m = 2; p = 2; n = 3;  
sys1 = drss(n, p, m);  
sys2 = ss2ss(sys1, rand(n));  
sys1 == sys2
```

however, it gives an error:

```
Operator '==' is not supported  
for operands of type 'ss'.
```

why are `ss` objects not comparable?

A system is a set of signals (the behavior)

signals

$w \in (\mathbb{R}^q)^{\mathbb{N}}, w : \mathbb{N} \rightarrow \mathbb{R}^q$	q -variate discrete-time signal
$w _T := (w(1), \dots, w(T))$	restriction of w to $[1, T]$
$\sigma, (\sigma w)(t) := w(t+1)$	unit shift operator

systems

$\mathcal{B} \subset (\mathbb{R}^q)^{\mathbb{N}}$	q -variables discrete-time system
$\mathcal{B} _T := \{w _T \mid w \in \mathcal{B}\}$	restriction of \mathcal{B} to $[1, T]$
$\mathcal{L}_{(m,\ell,n)}^q$	bounded complexity LTI systems

$\mathbf{m}(\mathcal{B}) / \mathbf{\ell}(\mathcal{B}) / \mathbf{n}(\mathcal{B})$ — # of inputs / lag / order

\mathcal{B} is represented by basis a B_T for $\mathcal{B}|_T$

$$\mathcal{B}|_T = \text{image} \begin{bmatrix} b^1 & \dots & b^r \end{bmatrix} \quad (B_T)$$

(B_T) is **nonparameteric** representation of $\mathcal{B}|_T$

$$w \in \mathcal{B}|_T \iff w = B_T g, g \in \mathbb{R}^r$$

$\mathcal{B} \in \mathcal{L}_c^q$ and $T \geq \ell(\mathcal{B})$ implies that

$$r = \dim \mathcal{B}|_T = \mathbf{m}(\mathcal{B})T + \mathbf{n}(\mathcal{B}) \quad (\dim \mathcal{B}|_T)$$

$\mathcal{B}|_{\ell(\mathcal{B})+1}$ defines $\mathcal{B} \implies (B_T)$ is **representation of \mathcal{B}**

B_T is obtained from data or representations

$$\text{data: } \mathcal{W}_d = \{w_d^1, \dots, w_d^N\} \mapsto B_T \quad (\text{w2BT})$$

$$(A, B, C, D) \mapsto B_T = \Pi \begin{bmatrix} 0_{mT \times n} & I_{mT} \\ \mathcal{O}_T(A, C) & \mathcal{I}_T(H) \end{bmatrix} \quad (\text{ss2BT})$$

$$\text{kernel: } R \mapsto B_T := \begin{bmatrix} b^1 & \dots & b^r \end{bmatrix} \quad (\text{R2BT})$$

indirect “data-driven approach” (B2BT)

1. simulate data \mathcal{W}_d from parametric representation
2. use w2BT to obtain (B_T) from the data

B_T is structured, due to the LTI dynamics

the structure is fully revealed when $T \geq \ell(\mathcal{B})$

$\mathcal{B}|_T$ is $(mT + n)$ -dimensional **shift-invariant subspace**

complexity bounded $\iff \dim \mathcal{B}|_T < qT$

$$\mathcal{B} \in \mathcal{L}_{(m,\ell,n)}^q \implies \dim \mathcal{B}|_T = mT + n = \text{rank } B_T$$

time-invariance \iff shift-invariance

- ▶ autonomous case: sum-of-exponentials
- ▶ open systems: the structure is hidden
- ▶ **image $\mathcal{H}_T(w_d)$ imposes shift-invariance by construction**

Computing B_T has hidden dangers



it involves rank computation

$$\dim \mathcal{B}|_T = qT - \text{rank} \mathcal{M}_T(R) = \text{rank} \mathcal{H}_T(\mathcal{W}_d)$$

rank is computed by singular values thresholding

$$\hat{r} := \# \text{ of singular values } \geq \text{tol} \quad (\hat{r})$$

the data has to be informative

$$\text{rank} \mathcal{H}_T(\mathcal{W}_d) = \mathbf{m}(\mathcal{B})T + \mathbf{n}(\mathcal{B}) \quad (\text{GPE})$$

The SVD approximation doesn't impose time-invariant structure on B_T

two options for complexity estimation:

1. specify tol , in which case r is found from (\hat{r}) , or
2. $c := (m, \ell, n)$, in which case r is found from $(\dim \mathcal{B}|_T)$

SVD approximation



*when “small” singular values are discarded,
 B_T is **not shift-invariant** and therefore
does not represent $\mathcal{B}|_T$ of $\mathcal{B} \in \mathcal{L}_c^q$*

Back to the example of systems equality

equality in the behavioral setting: $\mathcal{B}^1 \stackrel{?}{=} \mathcal{B}^2$

using (B_T) with $T \geq \max\{\ell(\mathcal{B}^1), \ell(\mathcal{B}^2)\}$

$$\mathcal{B}^1 = \mathcal{B}^2 \iff \mathcal{B}^1|_T = \mathcal{B}^2|_T$$

possible implementation

```
T = max(lag(B1), lag(B2));  
BT1 = B2BT(B1, T);  
BT2 = B2BT(B2, T);  
rank([BT1 BT2]) == rank(BT1)
```

analysis

- ▶ complexity computation
- ▶ input/output partitioning
- ▶ controllability test

BT2c
is_io, BT2IO
isunctr, distunctr

parametric representations

- ▶ kernel
- ▶ input/state/output
- ▶ conversions

BT2R, R2BT
BT2ss, ss2BT
R2ss, R2Rmin

identification and signal-processing

- ▶ exact identification
- ▶ approximate identification
- ▶ data-driven signal-processing

w2BT, w2R, w2ss
ident
ddint