A behavioral approach to direct data-driven fault detection

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The problem considered is to detect abnormal operation based on observed data

prior information about data-generating system

model-based vs direct data-driven methods

observed data collected offline and online

- dedicated experiment — known excitation signal
- “normal” operation — unknown excitation signal

We consider three data collection scenarios

free response / transient data

forced response with known excitation

forced response with unknown excitation

Outline

Free or forced response with known excitation

Forced response with unknown excitation

Empirical validation

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We view systems as sets of signals

$w \in (\mathbb{R}^q)^{\mathbb{N}}$ — q-variate discrete-time signal

$\mathscr{B} \subset (\mathbb{R}^q)^{\mathbb{N}}$ — q-variate dynamical model

- linear — $\mathscr{B}$ is a linear subspace of $(\mathbb{R}^q)^{\mathbb{N}}$
- time-invariant — invariant under shifts: $(\sigma w)(t) := w(t+1)$

$w \in \mathscr{B}$ means "w is a trajectory of $\mathscr{B}$"

In practice, we deal with finite signals

restriction of w / $\mathscr{B}$ to finite horizon $[1, T]$

$$w|_T := \big(w(1), \dots, w(T)\big), \quad \mathscr{B}|_T := \{\, w|_T \mid w \in \mathscr{B} \,\}$$

for $w_\mathrm{d} = \big(w_\mathrm{d}(1), \dots, w_\mathrm{d}(T_\mathrm{d})\big)$ and $1 \leq T \leq T_\mathrm{d}$

$$\mathscr{H}_T(w_\mathrm{d}) := \begin{bmatrix} (\sigma^0 w_\mathrm{d})|_T & (\sigma^1 w_\mathrm{d})|_T & \cdots & (\sigma^{T_\mathrm{d}-T} w_\mathrm{d})|_T \end{bmatrix}$$

$w_\mathrm{d} \in \mathscr{B}|_{T_\mathrm{d}}$ — “exact data”

The set of linear time-invariant systems $\mathscr{L}$ has structure characterized by integers

m — number of inputs

n — order ($=$ minimal state dimension)

ℓ — lag ($=$ observability index)

$\mathscr{L}_{(m,\ell,n)}$ — bounded complexity LTI systems

Nonparametric representation of LTI system's finite-horizon behavior

assumptions:

- $w_\mathrm{d} \in \mathscr{B}|_{T_\mathrm{d}}$ — exact offline data
- $\mathscr{B} \in \mathscr{L}_{(m,\ell,n)}$ — bounded complexity LTI system
- for $T \geq \boldsymbol{\ell}(\mathscr{B})$, $\operatorname{rank} \mathscr{H}_T(w_\mathrm{d}) = mT + n$ — informative data

then, the data-driven representation holds

$$\operatorname{image} \mathscr{H}_T(w_\mathrm{d}) = \mathscr{B}|_T \qquad \text{(DDR)}$$

The fault detection criterion is the distance from online data w to system's behavior $\mathscr{B}$

$\text{dist}(w, \mathscr{B}) := \min_{\widehat{w} \in \mathscr{B}|_T} \|w - \widehat{w}\|$

under the assumptions, using (DDR), we have

$$\text{dist}(w, \mathscr{B}) = \|w - \mathscr{H}_T(w_\text{d})\mathscr{H}_T^+(w_\text{d})w\|$$

direct data-driven computation of the distance

The fault detection method has offline and online steps

offline: using w_d, find orthonormal basis B for $\mathscr{B}|_T$

online: compute and threshold

$$\operatorname{dist}(w,\mathscr{B}) = \|(I - BB^\top)w\|$$

with noisy data w_d, the offline step is

- SVD truncation of $\mathscr{H}_T(w_\mathrm{d})$
- structured low-rank approximation of $\mathscr{H}_T(w_\mathrm{d})$
- model identification, using w_d

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With unobserved excitation signal e, prior knowledge about e is needed

zero-mean white Gaussian (disturbance)

deterministic signal $\leadsto$ input estimation problem

the model describes $w_{\text{ext}} := \left[\begin{smallmatrix} e \\ w \end{smallmatrix}\right]$

- e — unobserved signal
- w — observed signal

Finding e is a linear least-norm problem

given a model $\mathscr{B}_{\text{ext}}$ that describes $w_{\text{ext}} := \left[\begin{smallmatrix} e \\ w \end{smallmatrix}\right]$

$$\widehat{e}_{\text{ln}} := \arg\min_{(\widehat{e},w)\in\mathscr{B}_{\text{ext}}|_T} \ \|\widehat{e}\|$$

exact recovery $\widehat{e}_{\text{ln}} = e$ is not possible

Deterministic input estimation is linear least-squares problem

Π_e / Π_w — projection of $w_{\text{ext}} := \left[\begin{smallmatrix} e \\ w \end{smallmatrix}\right]$ on e / w

given, $\widehat{\mathscr{B}}_{\text{ext}}|_T = \operatorname{image} B_{\text{ext}}$ (basis for $\widehat{\mathscr{B}}_{\text{ext}}|_T$)

$\widehat{e} := \Pi_e B_{\text{ext}} (\Pi_w B_{\text{ext}})^+ w$

Fault detection method with unobserved input

generalized distance measure:

$$\mathrm{dist}(w, \mathscr{B}_{\mathrm{ext}}) := \min_{(\widehat{e}, \widehat{w}) \in \mathscr{B}|_T} \|w - \widehat{w}\|$$

offline: using $(e_{\mathrm{d}}, w_{\mathrm{d}})$, find basis B_{ext} for $\mathscr{B}_{\mathrm{ext}}|_T$ and let $B_w := \Pi_w B_{\mathrm{ext}}$

online: compute and threshold

$$\mathrm{dist}(w, \mathscr{B}_{\mathrm{ext}}) = \left\|(I - B_w B_w^\top) w\right\|$$

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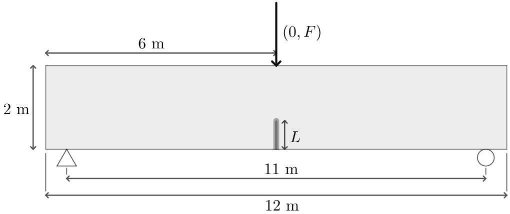
Validation on vibrating beam with crack subject to unobserved disturbance force



data w_d^k	crack length	loss of stiffness	type of damage
0	0.0m	0%	none
1	0.7m	100%	severe
2	0.7m	36%	medium
3	0.2m	100%	medium
4	0.2m	36%	mild

observed displacements left / right of the crack

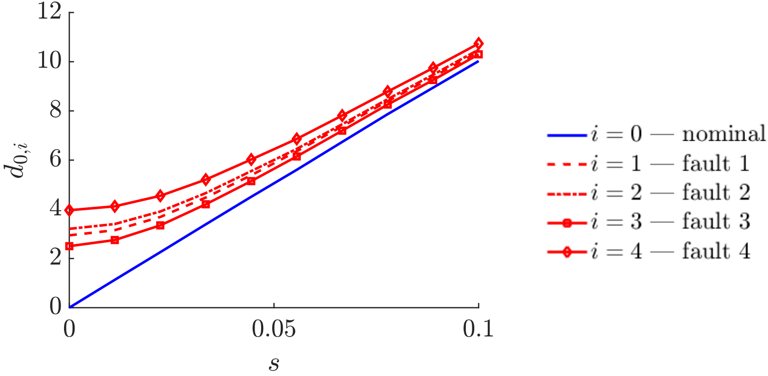
hyper-parameters: $T = 100$, $\ell = 2$, $n = 6$

offline computation: $\mathscr{B}^k$ using w_{d}^k

online computation: $d_{0,k} := \mathrm{dist}(w^0, \mathscr{B}^k)$

noise with standard deviation s added to w^0

Distances from nominal data to models as function of noise level



Comments

the beam behaves like 6th order LTI system

most severe crack is not hardest to detect

effect of the sensor location

Outlook

assumptions:

- bounded complexity LTI system
- hyper-parameters: horizon T and lag ℓ
- different ways to deal with noise in offline data w_{d}

advantages:

- representation invariant distance measure
- can deal with unobserved disturbance signal
- cheap to compute online and simple to implement

other applications