# Finite-data nonparametric frequency response evaluation without leakage

I. Markovsky and H. Ossareh

# Problem formulation

given: "data" trajectory $(u_\mathrm{d}, y_\mathrm{d}) \in \mathscr{B}|_{T_\mathrm{d}}$ and $z \in \mathbb{C}$

find: $H(z)$, where $H$ is the transfer function of $\mathscr{B}$

# Direct data-driven solution

we are interested in trajectory

$$w = \begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} \exp_z \\ \widehat{H} \exp_z \end{bmatrix} \in \mathscr{B}, \quad \text{where } \exp_z(t) := z^t$$

using the data-driven representation, we have

$$\begin{bmatrix} \mathscr{H}_L(u_\mathrm{d}) \\ \mathscr{H}_L(y_\mathrm{d}) \end{bmatrix} g = \begin{bmatrix} \mathbf{z} \\ \widehat{H}\mathbf{z} \end{bmatrix}, \quad \text{where } \mathbf{z} := \begin{bmatrix} z^1 \\ \vdots \\ z^L \end{bmatrix}$$

which leads to the system

$$\begin{bmatrix} 0 & \mathscr{H}_L(u_\mathrm{d}) \\ -\mathbf{z} & \mathscr{H}_L(y_\mathrm{d}) \end{bmatrix} \begin{bmatrix} \widehat{H} \\ g \end{bmatrix} = \begin{bmatrix} \mathbf{z} \\ 0 \end{bmatrix} \tag{SYS}$$

# Solution method: solve (SYS) for $\widehat{H}$

under GPE condition, with $L \geq \ell + 1$, $\widehat{H} = H(z)$

trivial generalization to

- multivariable systems
- multiple data trajectories $\{ w_{\mathrm{d}}^1, \dots, w_{\mathrm{d}}^N \}$
- evaluation of $H(z)$ at multiple points in $\{ z_1, \dots, z_K \} \in \mathbb{C}^K$

# Comparison with classical nonparametric frequency response estimation methods

ignored initial/terminal conditions $\rightsquigarrow$ *leakage*

DFT grid $\rightsquigarrow$ limited *frequency resolution*

improvements by windowing and interpolation

- the leakage is not eliminated
- the methods involve *hyper-parameters*

# Generalization of (SYS) to noisy data

## preprocessing: rank-$mL+n$ approx. of $\mathscr{H}_L(w_\text{d})$

- hyper-parameters $L \geq \ell+1$ and $n$
- if the approximation preserves the Hankel structure, the method is maximum-likelihood in the EIV setting

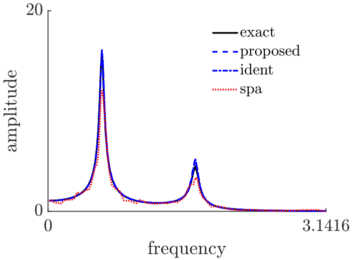
## regularization with $\|g\|_1$

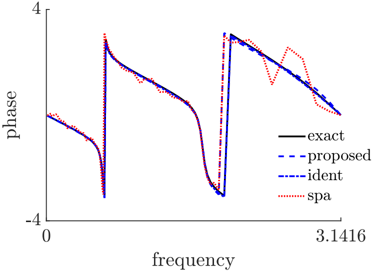
- hyper-parameter: the 1-norm regularization parameter

# Example: EIV setup with 4th order system

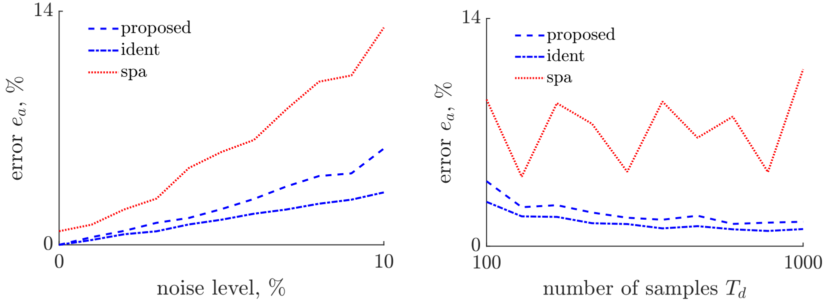
## the proposed method is compared with

- `ident` — parametric maximum-likelihood estimator
- `spa` — nonparameteric estimator with Welch filter



# Monte-Carlo simulation over different noise levels and number of samples




$$e_a := \frac{||\overline{H}_z| - |\widehat{H}_z||}{|\overline{H}_z|}\ 100\%$$

# Publication and code

- I. Markovsky and H. Ossareh.
  Finite-data nonparametric frequency response evaluation without leakage. Automatica, 159:111351, 2024.

- `https://imarkovs.github.io/frest`