Dynamic measurement: application of system identification in metrology

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Dynamic measurement takes into account the dynamical properties of the sensor

model of sensor as dynamical system

to-be-measured variable u $\xrightarrow{\text{measurement process}}$ measured variable y

assumptions

1. measured variable is constant $u(t) = \bar{u}$
2. the sensor is stable LTI system
3. sensor's DC-gain $= 1$ (calibrated sensor)

I can't understand anything in general unless I'm carrying along in my mind a specific example and watching it go. *R. Feynman*

examples of sensors:

1. thermometer
2. weighing scale

Thermometer is 1st order dynamical system

$$\begin{array}{ccc} \text{environmental} & \xrightarrow{\text{heat transfer}} & \text{thermometer's} \\ \text{temperature } \bar{u} & & \text{reading } y \end{array}$$

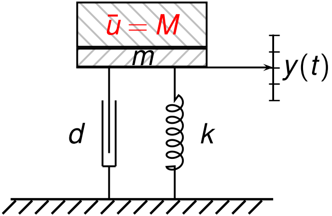
measurement process: Newton's law of cooling

$$\dot{y} = a\left(\bar{u} - y\right)$$

the heat transfer coefficient $a > 0$ is in general unknown

DC-gain $= 1$ is a priori known

Scale is 2nd order dynamical system

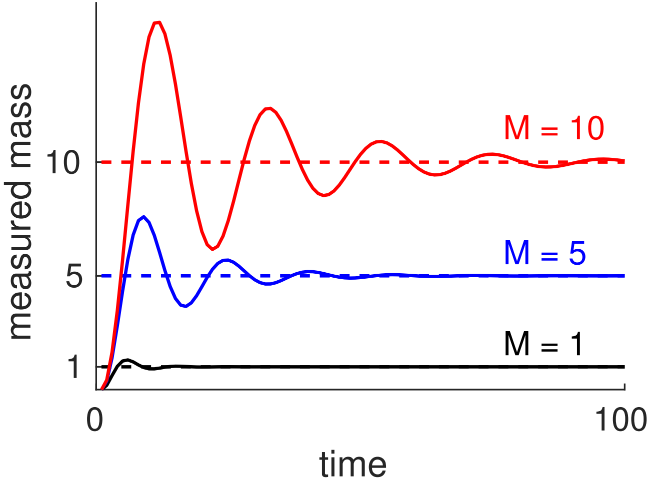


$$(M+m)\ddot{y} + d\dot{y} + ky = g\bar{u}$$

process dynamics depends on $M \quad \Longrightarrow \quad$ unknown

DC-gain $= g/k$ — known for given scale (on the Earth)

Measurement process dynamics depends on the to-be-measured mass



Sensor's transient response contributes to the measurement error

transient decays exponentially

however measuring longer is undesirable

main idea: predict the steady-state value

Plan

Dynamic measurement state-of-the-art

Model-based maximum-likelihood estimator

Data-driven maximum-likelihood estimator

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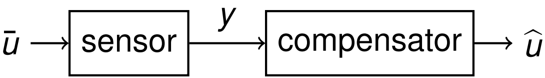
Classical approach of design of compensator



goal: find a compensator, such that $\widehat{u} = \bar{u}$

idea: use the inverse system $C = S^{-1}$, where

- S is the transfer function of the sensor
- C is the transfer function of the compensator

Inverting the model is not a general solution

1. S^{-1} may not exist / be a non-causal system
2. initial conditions and noise on y are ignored
3. the sensor dynamics has to be known

Modern approach of using adaptive signal processing

real-time compensator tuning

requires real-time model identification

solutions specialized for 2nd order processes

W.-Q. Shu. Dynamic weighing under nonzero initial conditions. IEEE Trans. Instrumentation Measurement, 42(4):806–811, 1993.

There are opportunities for SYSID community to contribute

ad-hock methods

restricted to 1st / 2nd order SISO processes

lack of general approach and solution

Dynamic measurement is non standard SYSID problem

of interest is the steady-state $\bar{u}$ (not the model)

the input is unknown (blind identification)

the DC-gain is a priori known

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Data-driven maximum-likelihood estimator

The data is generated from LTI system with output noise and constant input

$$\underbrace{y_{\mathrm{d}}}_{\substack{\text{measured}\\\text{data}}} = \underbrace{y}_{\substack{\text{true}\\\text{value}}} + \underbrace{e}_{\substack{\text{measurement}\\\text{noise}}}$$

$$\underbrace{y}_{\substack{\text{true}\\\text{value}}} = \underbrace{\bar{u}}_{\substack{\text{steady-state}\\\text{value}}} + \underbrace{y_0}_{\substack{\text{transient}\\\text{response}}}$$

assumption 4: e is a zero mean, white, Gaussian noise using state space representation of the sensor

$$x(t+1) = Ax(t), \qquad x(0) = x_0$$
$$y_0(t) = cx(t)$$

we obtain

$$\underbrace{\begin{bmatrix} y_\mathrm{d}(1) \\ y_\mathrm{d}(2) \\ \vdots \\ y_\mathrm{d}(T) \end{bmatrix}}_{y_\mathrm{d}} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{\mathbf{1}_T} \bar{u} + \underbrace{\begin{bmatrix} c \\ cA \\ \vdots \\ cA^{T-1} \end{bmatrix}}_{\mathscr{O}_T} x_0 + \underbrace{\begin{bmatrix} e(1) \\ e(2) \\ \vdots \\ e(T) \end{bmatrix}}_{e}$$

Maximum-likelihood model-based estimator

solve approximately

$$\begin{bmatrix} \mathbf{1}_T & \mathscr{O}_T \end{bmatrix} \begin{bmatrix} \widehat{u} \\ \widehat{x}_0 \end{bmatrix} \approx y_\mathrm{d}$$

standard least-squares problem

$$\begin{array}{ll} \text{minimize} & \text{over } \widehat{y}, \widehat{u}, \widehat{x}_0 \quad \|y_\mathrm{d} - \widehat{y}\| \\ \text{subject to} & \begin{bmatrix} \mathbf{1}_T & \mathscr{O}_T \end{bmatrix} \begin{bmatrix} \widehat{u} \\ \widehat{x}_0 \end{bmatrix} = \widehat{y} \end{array}$$

recursive implementation $\rightsquigarrow$ Kalman filter

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Subspace model-free method

goal: avoid using the model parameters (A, C, $\mathscr{O}_T$)

in the noise-free case, due to the LTI assumption,

$$\Delta y(t) := y(t) - y(t-1) = y_0(t) - y_0(t-1)$$

satisfies the same dynamics as y_0, *i.e.*,

$$\begin{aligned} x(t+1) &= Ax(t), \qquad x(0) = \Delta x \\ \Delta y(t) &= cx(t) \end{aligned}$$

if Δy is persistently exciting of order $\mathtt{n}$

$$\operatorname{image}(\mathscr{O}_{T-\mathtt{n}}) = \operatorname{image}\big(\mathscr{H}(\Delta y)\big)$$

where

$$\mathscr{H}(\Delta y) := \begin{bmatrix} \Delta y(1) & \Delta y(2) & \cdots & \Delta y(\mathtt{n}) \\ \Delta y(2) & \Delta y(3) & \cdots & \Delta y(\mathtt{n}+1) \\ \Delta y(3) & \Delta y(4) & \cdots & \Delta y(\mathtt{n}+2) \\ \vdots & \vdots & & \vdots \\ \Delta y(T-\mathtt{n}) & \Delta y(T-\mathtt{n}) & \cdots & \Delta y(T-1) \end{bmatrix}$$

model-based equation

$$\begin{bmatrix} \mathbf{1}_T & \mathscr{O}_T \end{bmatrix} \begin{bmatrix} \bar{u} \\ \widehat{x}_0 \end{bmatrix} = y$$

data-driven equation

$$\begin{bmatrix} \mathbf{1}_{T-\mathtt{n}} & \mathscr{H}(\Delta y) \end{bmatrix} \begin{bmatrix} \bar{u} \\ \ell \end{bmatrix} = y|_{T-\mathtt{n}} \qquad (*)$$

subspace method: solve $(*)$ by (recursive) least squares

The subspace method is suboptimal

subspace method

$$\text{minimize} \quad \text{over } \widehat{y}, \widehat{u}, \widehat{\ell} \quad \| y_d|_{T-\mathtt{n}} - \widehat{y} \|$$
$$\text{subject to} \quad \begin{bmatrix} \mathbf{1}_{T-\mathtt{n}} & \mathscr{H}(\Delta y_d) \end{bmatrix} \begin{bmatrix} \widehat{u} \\ \widehat{\ell} \end{bmatrix} = \widehat{y}$$

maximum likelihood model-free estimator

$$\text{minimize} \quad \text{over } \widehat{y}, \widehat{u}, \widehat{\ell} \quad \| y_d|_{T-\mathtt{n}} - \widehat{y} \|$$
$$\text{subject to} \quad \begin{bmatrix} \mathbf{1}_{T-\mathtt{n}} & \mathscr{H}(\Delta \widehat{y}) \end{bmatrix} \begin{bmatrix} \widehat{u} \\ \widehat{\ell} \end{bmatrix} = \widehat{y}$$

structured total least-squares problem

Summary

dynamic measurement is identification(-like) problem

however, the goal is to estimate the stead-state value

ML estimation $\rightsquigarrow$ structured total least squares

Perspectives

recursive solution of the STLS problem

statistical analysis of the subspace method

generalization to non-constant input