On the Conditional Score and Corrected Score Estimation in Nonlinear Measurement Error Models

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Abstract. This paper reviews the conditional score and corrected score estimation of the unknown parameters in nonlinear measurement error (errors-in-variables) models. This includes the functional and structural models. The connection among these methodologies and total least squares (TLS) is also examined. A compendium of existing results as well as some possible extensions are discussed.

The ordinary regression models assume that the independent variables are measured without error. However, in many situations, the independent variables cannot measured precisely. When the measurement error is too large to ignore, the estimators for the regression parameters are biased and inconsistent. Measurement error models are important alternatives for ordinary regression models, in which we assume that relation between the independent variable y and independent variable $\boldsymbol{\xi}$ is known but one cannot observe $\boldsymbol{\xi}$ directly. Instead, one observes $\mathbf{x} = \boldsymbol{\xi} + \boldsymbol{\delta}$, where $\boldsymbol{\delta}$ is independents of $\boldsymbol{\xi}$ and has mean zero.

The linear measurements error model has a long history and is dated back 1877 (Adcock, 1977), which has been well investigated. For a summary, see Fuller (1987) and Cheng and Van Ness (1999). For the past two decades, the researches on measurement error models are more focused on nonlinear measurement error models, see Carroll, Ruppert and Stefanski (1995) for a reference.

There are two general methodologies proposed in the literature to estimate the regression parameters in nonlinear measurement error models. The first one is the *conditional score* method that was proposed by Stefanski and Carroll

(1987). The second one is called *corrected score* method, which was proposed by Stefanski (1989) and Nakamura (1990) independently.

In this paper, we will review these two methods. In our view, they have some fundamental difference in their assumptions that has been neglected in the literature. We will also bring some recent developments to attention and some possible extensions are discussed. Finally the connection between the conditional score method and TLS (total least squares) is addressed.

References

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