Some issues on Errors-in-Variables Identification

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The proposed paper would concern the topics described in the sequel.

The EIV context

The Errors-in-Variables context is a challenging environment well known from many years that has seen an increasing amount of research and, consequently, of new results, only in relatively recent times. One of the appealing features of EIV models consists in their intrinsic capability of describing real processes and in relying only on limited sets of *a–priori* assumptions [24, 25]. These features suggest the use of EIV models in all applications like, for instance, diagnosis, where the interest is focused on a realistic description of a process more than on other features like prediction.

The Frisch scheme

The scheme proposed by the Nobel prize Ragnar Frisch in 1934 [3] is an interesting compromise between the great generality of the EIV environment and the possibility of real applications. Moreover, the Frisch scheme encompasses some other important schemes like Least Squares and the Eigenvector Method and plays, consequently, a role of paramount importance also from a conceptual point of view. The compatibility of the Frisch scheme with a whole family of solutions has diverted the attention towards simpler schemes leading to single solutions.

Loci of solutions and their properties in the algebraic case

Any analysis of the Frisch scheme cannot ignore the existence of two separate loci of solutions, one in the parameter space and the other in the space of the variances of the noise affecting the considered variables and, even more important, the maps between these loci. Some fundamental results [7, 8] describe these maps as well as the shape of the loci in the parameter space under specific conditions (inverse of the covariance matrix of the noisy data Frobenius–like) [2]. Unfortunately the locus of solutions in the parameter space can be easily defined only when the data are compatible with a single linear relation; in all other cases the reference to the parameter space does not lead to significant results. The investigation of the properties of the locus of solutions in the noise space has offered the key for a deeper analysis that shows how this locus (a convex hypersurface lying in the first orthant), differently from what happens in the parameter space, does not degenerate in any case and enjoys some other important properties [4, 6].

The maximum corank problem

One of the problems considered of paramount importance in the ecometric field consists in detrmining the maximal number of linear relations compatible with a given set of noisy data. The importance attributed to this problem is due to the fact that econometricians consider the solution of this problem as linked to the extraction of the whole information contained in the data [9]. The solution of this problem in the context of the Frisch scheme has been possible only making reference to the properties of the locus of solutions in the noise space [5]; other approaches have led to determine an upper bound to this number [10].

Relations between algebraic and dynamic contexts

When the data are generated by a dynamic system and the Frisch context is used for its identification, it is necessary to consider the properties of the loci of solutions under the constraints imposed by the shift properties of dynamic systems [13]. It can a bit surprising to discover that, in this respect, the dynamic case can be seen as a subcase of the algebraic one and that the previously mentioned shift properties lead (in general) to a unique solution [1, 11, 23]. It can also be surprising to show how this solution is linked to the solution of the maximal corank problem in the algebraic case.

EIV schemes and real data

All previous considerations refer to an exact fullfillment of the assumptions behaind the Frisch scheme (noise whiteness etc.) that could be satisfied, at best, only in asymptotic conditions. In all practical cases this cannot be achieved not even asyptotically because of a whole series of violations due to non linearity, non stationarity etc. The development of Frisch identification procedures requires thus the introduction of suitable criteria leading to the selection of a single model [14, 15, 16].

Bias Eliminated Least Squares and Instrumental Variable methods

The Frisch scheme in the identification of dynamic processes enjoys some peculiarities like the congruence between the model and the estimated noise variances but does not constitute the only practical way for solving this problem. Another appealing method is based on the elination of the bias that would be present by applying least squares. BELS methods constitute a large family of fast algorithms that, even if affected by convergence problems, can give good results [29, 27, 28]. The very general applicability context of IV methods allows also their use in solution of EIV identification problems [19, 20, 26]. Despite their simple implementation and stimulating asymptotic properties the methods are affected by large estimation covariance with limited sets of data [22, 21].

Maximum Likelihood approaches

When the ratio of the noise variances is *a–priori* known or when the input can be described by means of an ARMA process, it is possible, when the noise distribution is known, to apply a ML approach [12, 18, 17]. These approaches lead, in previous contexts, to the best achievable accuracy but can be affected by problems of convergence to local minima.

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