Frequency Domain Maximum Likelihood Estimation of Linear Dynamic Errors-in-Variables Models

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Abstract - This paper studies the linear dynamic errors-in-variables problem in the frequency domain. First the identifiability is shown under relaxed conditions. Next a frequency domain Gaussian maximum likelihood (ML) estimator is constructed that can handle discrete-time as well as continuous-time models on (a) part(s) of the unit circle or imaginary axis. The ML estimates are calculated via a computational simple and numerical stable Newton-Gauss minimization scheme. Finally the Cramér-Rao lower bound is derived.

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1. Introduction

Linear dynamic errors-in-variables (EIV) modelling is important in those applications where one is looking after a better understanding of the underlying input-output relation of a process rather than making an output prediction from noisy observations. One can distinguish between two cases: either the excitation of the process can freely be chosen, or one has to live with the operational (natural) perturbations. If the excitation can freely be chosen then it is strongly recommended to use periodic excitation signals because it simplifies significantly the identification problem: (i) nonparametric estimates of the disturbing noise (co-)variances are obtained in a preprocessing step, and (ii) since mutually correlated, coloured input/output errors are allowed, identification in feedback is just a special case of the general framework (see Pintelon and Schoukens, 2001). In the second case the excitation is often random and parts of it may even be unmeasurable. This paper handles the second case, assuming that the excitation is a stochastic process with rational power spectrum. As will be shown in the sequel of the paper the second case is much more complicated than the first: besides the plant model one should also identify simultaneously the signal, and the input/output noise models.

Identifiability is a first key issue in EIV modelling: under which conditions on the excitation, the input/output errors, and the process is the EIV problem uniquely solvable? This question has been studied in detail in econometrics and an extensive literature is available (see Söderström, 2006 for an exhaustive overview). For example, Anderson and Deistler (1984) handles the identifiability of scalar EIV problems with coloured input/output errors, while Nowak (1993) covers the multivariable case. A second key issue is the numerical calculation of the EIV estimates. Several algorithms have been proposed, each of them having their specific advantages and disadvantages (see Söderström, 2006 for an exhaustive overview). For example, spectral factorization is the computational bottle neck of the statistically efficient time domain maximum likelihood method (Söderström and Stoica, 1989), while the computational simple instrumental variable methods have low statistical accuracy (Söderström, 2006). Except for Mahata and Garnier (2005), all methods handle the discrete-time case and no algorithms for direct continuous-time EIV modelling are available. In Mahata and Garnier (2005) a method is presented for identifying continuous-time models from non-uniformly sampled data in the presence of white input/output errors.

The contributions of this paper are:

1. the identifiability of general linear dynamic EIV models is shown under relaxed conditions,
2. a (computational simple) frequency domain Gaussian maximum likelihood (ML) estimator is developed for the general case of coloured and mutually independent input/output errors,
3. the ML estimator can handle discrete-time as well as continuous-time modelling on (a) part(s) of the unit circle or imaginary axis,
4. a numerical stable Newton-Gauss minimization scheme of the ML cost function is derived,
5. easy calculation of the Cramér-Rao lower bound.

2. References

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