Applications of TLS and Related Methods in the Environmental Sciences

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Abstract

Rainfall-Runoff and Signal Separation Problems: The process of converting rainfall into runoff is a highly nonlinear problem due to the soil-water interaction that starts when rainfall reaches the ground. Additional variables to consider are evaporation, transpiration, losses due to vegetation and land use, and the different flow processes that take place in a watershed. For instance, baseflow is a much slower process than groundwater and surface flow. Given records of rainfall and runoff data, one can build an accurate state-space model such as

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + Du_k + v_k, \end{aligned}$$

where at time k, u_k, y_k, and x_k are, respectively, the rainfall, runoff, and the state of the system. Such models have been used in real-time forecasting scenarios for flood control purposes [12]. However, the above model does not take into account the nonlinearities of the rainfall-runoff process. Most lumped rainfall-runoff models separate the baseflow and groundwater components from the measured runoff hydrograph in an attempt to model these as linear hydrologic reservoir units. Similarly, rainfall losses due to infiltration as well as other abstractions are separated from the measured rainfall hyetograph, which are then used as inputs to the linear hydrologic reservoir units. This data pre-processing is in essence a nonlinear signal separation problem that separates rainfall into infiltration and excess rainfall, and the measured hydrograph into surface flow and groundwater flow. These are then used to build separate linear models such as

$$\begin{aligned} x_{k+1}^g &= A_g x_k^g + B_g u_k^g \\ y_k^g &= C_g x_k^g + D_g u_k^g, \end{aligned}$$

$$\begin{aligned} x_{k+1}^s &= A_s x_k^s + B_s u_k^s \\ y_k^s &= C_s x_k^s + D_s u_k^s, \end{aligned}$$

where

$$\begin{aligned} u_k &= u_k^g + u_k^s \\ y_k &= y_k^g + y_k^s. \end{aligned}$$

In the separation process, a TLS approach is used since the infiltration process is an exponential signal. Thus, the classical NMR fitting techniques [2, 6, **?**, 17] are used.

Physical Parameter Extraction Problems: When modeling physical processes such as infiltration, where water flows into different compartments, one is faced with a physical parameter extraction problem. This is quite evident in black-box system identification where an unknown similarity transformation matrix destroys the physical meaning of the problem. Here we show that such similarity trsnsformation can be recovered as a post identification TLS problem. That is, suppose the identified state-space system matrices are $\{\bar{A}, \bar{B}, \bar{C}, \bar{D}\}$, while the physical parameter matrices are those of a mass-spring-damper system with mass m, spring constant k, and damping coefficient b. The table below shows the parameter matrices.

Physical Model	Identified Model
$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}$	$\bar{A}_c = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$
$B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$	$\bar{B}_c = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$
$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$	$\bar{C}_c = \begin{bmatrix} c_{11} & c_{12} \end{bmatrix}$
$D = \begin{bmatrix} 0 \end{bmatrix}$	$\bar{D}_c = \begin{bmatrix} 0 \end{bmatrix}$

The two systems are related by a similarity transformation T, i.e., $T\bar{A}T^{-1} = A$, $T\bar{B} = B$, and $\bar{C}T^{-1} = C$. As one can see, this system of equations is nonlinear, but if we re-write it as $T\bar{A} = AT$, $T\bar{B} = B$, and $\bar{C} = CT$, then we convert the problem into a linear one. It turns out that the solution can be framed as an orthogonal complement problem of the form

$$\begin{bmatrix} t_{11} & t_{12} & t_{21} & t_{22} & M & N & Z & -1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & b_{11} & 0 & -1 & 0 \\ a_{21} & a_{22} & 0 & 0 & b_{21} & 0 & 0 & -1 \\ -1 & 0 & a_{11} & a_{12} & 0 & b_{11} & 0 & 0 \\ 0 & -1 & a_{21} & a_{22} & 0 & b_{21} & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_{11} & c_{12} \end{bmatrix} = 0_{1\times 8},$$

or

$$x^T \mathcal{A} \ = \ 0_{1\times 8},$$

where

$$\begin{aligned} Z &= \frac{1}{m} \\ M &= -\frac{k}{m}t_{11} - \frac{b}{m}t_{21} \\ N &= -\frac{k}{m}t_{12} - \frac{b}{m}t_{22}. \end{aligned}$$

We will generalize the above results and show an example of a two-tank reservoir model.

Other Applications and Related Methods: We will also discuss applications of TLS in hyperspectral analysis, variogram fitting of spatial processes, and Chemometrics applications in the environemental sciences.

KEYWORDS: Horton's infiltration model, hydrograph separation, variogram fitting, exponential data fitting, singular value decomposition, total least squares, nonlinear least squares, and state-space models, physical model identification, hyperspectral analysis, partial least squares.

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About the author

José Ramos obtained his BSCE from the University of Puerto Rico at Mayaguez in 1979. He then went to Georgia Institute of Technology and completed his MS and Ph.D. degrees in 1979 and 1985, respectively. From 1985 to 1990 he worked at United Technologies Optical Systems in West Palm Beach, Florida, where he developed Kalman filtering and tracking algorithms as part of a military program on adaptive optics for space applications. That same year joined the ESAT goup at the Katholieke Universiteit Leuven as a post doctoral fellow, working on subspace system identification algorithms for linear, bilinear, and 2-D systems. From 1991 to 1993 he was a post doctoral fellow at the Institute for Land and Water Management, Katholieke Universiteit Leuven. He has been at Indiana University Purdue University - Indianapolis since 1997, where he teaches courses in stochastic processes, instrumentation, modern control, circuit theory, system identification, and system theory. His current research interests are in the areas of system identification, multivariate data analysis, optimization, and applied numerical linear algebra. His applications are in the areas of water resources, biomedical signal processing, image registration, and time series analysis. He has collaborated with The University of Montpellier II on the use of splines in nonlinear system identification, and with the University of Oporto on various iterative subspace system identification algorithms.