COVENTRY UNIVERSITY

CTAC
Control Theory and Applications Centre

Fourth Total Least Squares and Errors-in-Variables Modelling Workshop

Model-Based Control in the Errors-in-Variables Framework

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Control Theory and Applications Centre
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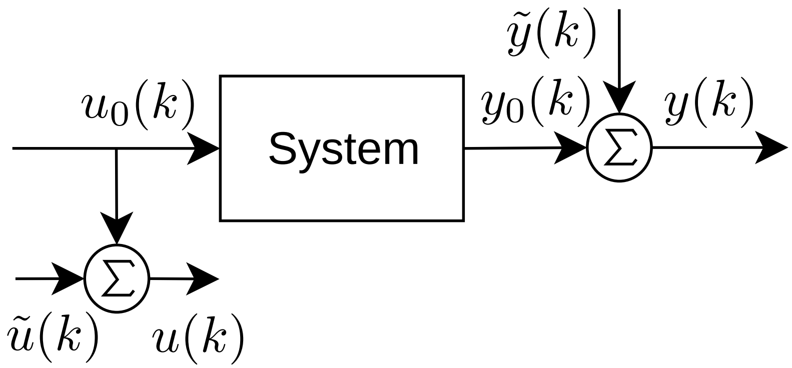
Plan

- A modified EIV setup for control
- Model based control in the EIV framework
- EIV-Kalman filter/EIV-extended Kalman filter
- Simulation results
- Conclusive remarks & further work

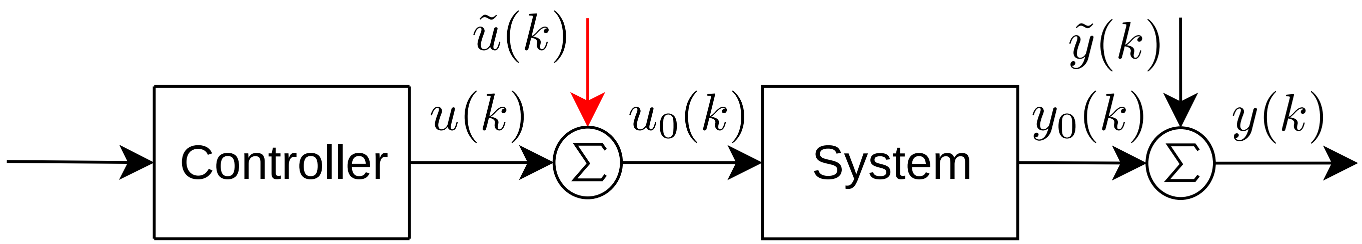
A modification of the EIV-setup

- Classical EIV-setup:



$u_0(k)$:	noise free input
$y_0(k)$:	noise free output
$\tilde{u}(k)$:	input measurement noise
$\tilde{y}(k)$:	output measurement noise
$u(k)$:	available system input
$y(k)$:	available system output

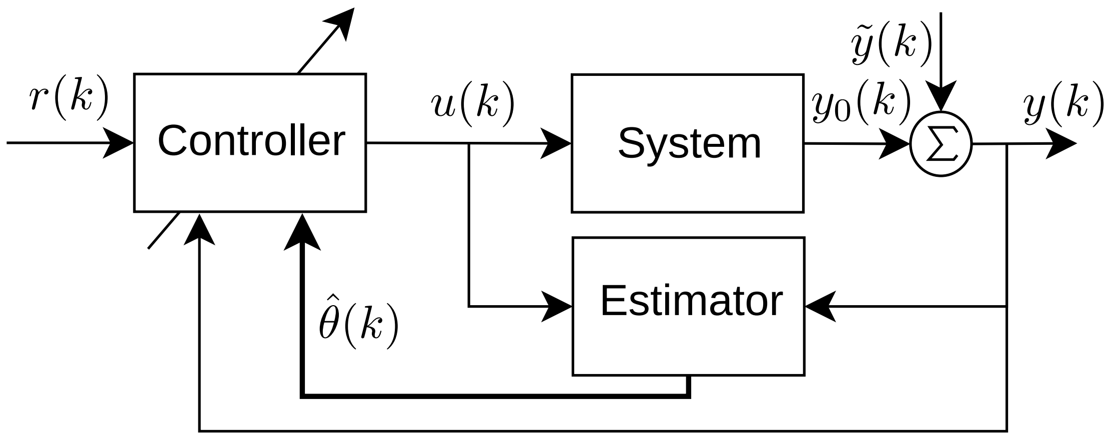
- Modified EIV-setup: control action $u(k)$ assumed to be known, $u_0(k)$ unknown



$\tilde{u}(k)$:	unobserved input
$u_0(k)$:	true system input

Classical model-based control setup

- Model-based control:

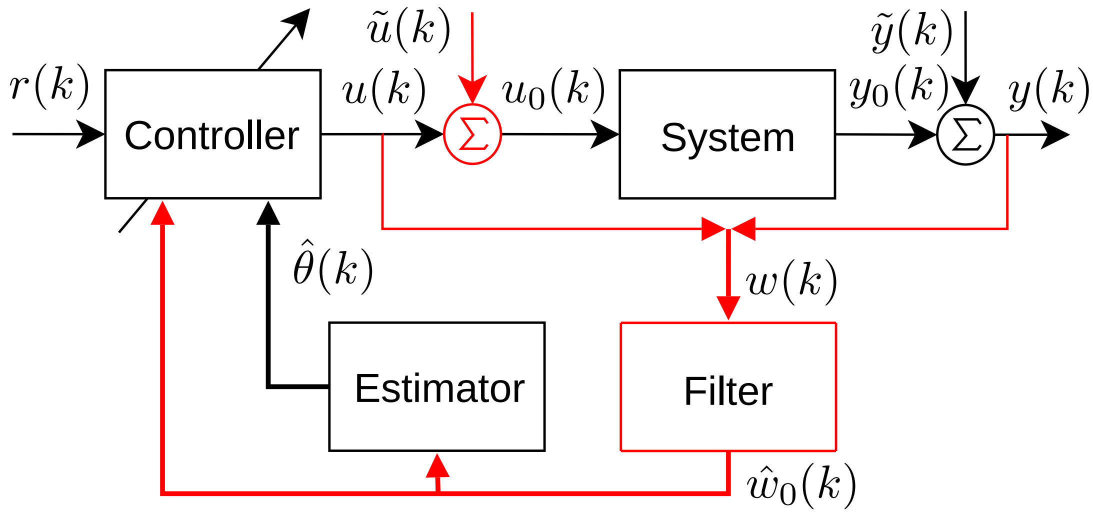


$r(k)$	reference signal
$u(k)$	control action/available input
$y_0(k)$	true system output
$\tilde{y}(k)$	output noise (measurement, model mismatch)
$\hat{\theta}(k)$	online estimate of system parameters θ

Model based control in the EIV framework

- Objective: Obtain estimates of $u_0(k)$ and $y_0(k)$ for the purpose of model based control

- Model-based control in the EIV framework:



$\tilde{u}(k)$	unobserved input
$u_0(k)$	true system input

$$w(k) := \begin{bmatrix} y(k) \\ u(k) \end{bmatrix}, \hat{w}_0(k) := \begin{bmatrix} \hat{y}_0(k) \\ \hat{u}_0(k) \end{bmatrix}$$

Closed-loop considerations

- Controllers:

  1. PID controller
  2. General Minimum Variance Controller (GMVC), l-step ahead controller, minimises cost function

$$V(P,Q,R) = E\left[\left[Py\left(k+l\right) - Rr(k)\right]^2 + \left[Qu(k)\right]^2\right] \tag{1}$$

- Estimator: Recursive least squares (RLS) estimator

- Filters:

  1. Kalman filter (Kf)
  2. EIV-Kalman filter (EIV-Kf) (*Guidorzi et al. 2002*)
  3. EIV-extended Kalman filter (EIV-eKf) *(Vinsonneau et al. 2005)*

EIV-Kalman filter (EIV-Kf) (Guidorzi *et al.* 2002)

- Estimate noise components $\tilde{u}(k)$, $\tilde{y}(k)$, hence providing estimate $\hat{w}_0(k)$

- Assumptions:
  - Linear time-invariant (LTI) system with known parameter θ
  - Additive input/output white noise sequences with known covariance matrix

$$\Sigma_{\tilde{y}\tilde{u}} = \begin{bmatrix} \sigma_{\tilde{y}} & \sigma_{\tilde{y}\tilde{u}} \\ \sigma_{\tilde{u}\tilde{y}} & \sigma_{\tilde{u}} \end{bmatrix} \tag{2}$$

with

$$\tilde{\sigma}_y = E[\tilde{y}^2(k)], \qquad \tilde{\sigma}_u = E[\tilde{u}^2(k)], \qquad \tilde{\sigma}_{uy}^T = \tilde{\sigma}_{yu} = E[\tilde{y}(k)\tilde{u}(k)] \tag{3}$$

$\sigma_{\tilde{y}}$, $\sigma_{\tilde{u}}$, are mutually correlated, uncorrelated with $u_0(k)$

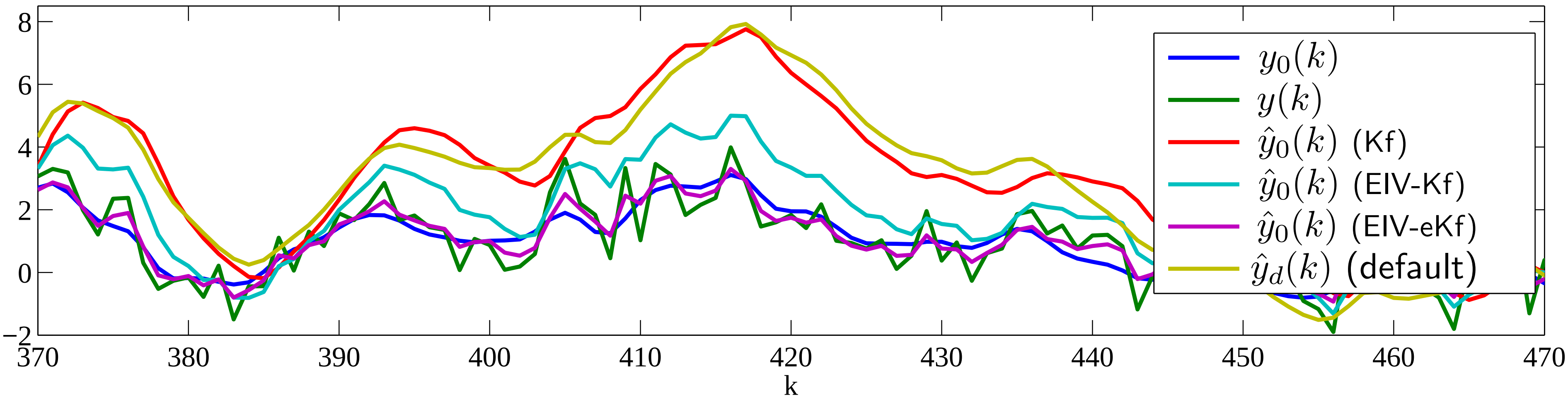
- Here modified for unity delay in control systems

$$u(k) \rightarrow u(k-1), \qquad \tilde{u}(k) \rightarrow \tilde{u}(k-1) \tag{4}$$

EIV-extended Kalman filter (EIV-eKf) (Vinsonneau *et al.* 2005)

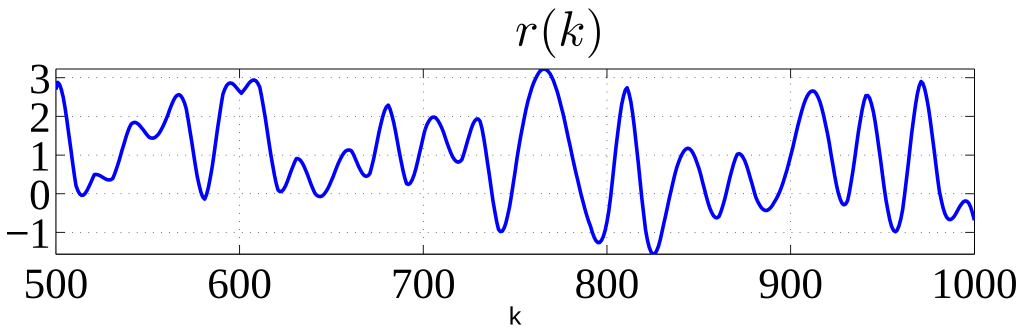
- Extension of EIV-Kf $\rightarrow$ EIV-eKf $\neq$ eKf for EIV in poster
  - Relaxation of LTI assumption $\rightarrow$ ability to handle linear time-varying (LTV) systems and/or model mismatch
  - Utilises default linear model characterised by $\theta_d \neq \theta$

- Typical illustrative example:



Simulation study: a SISO case

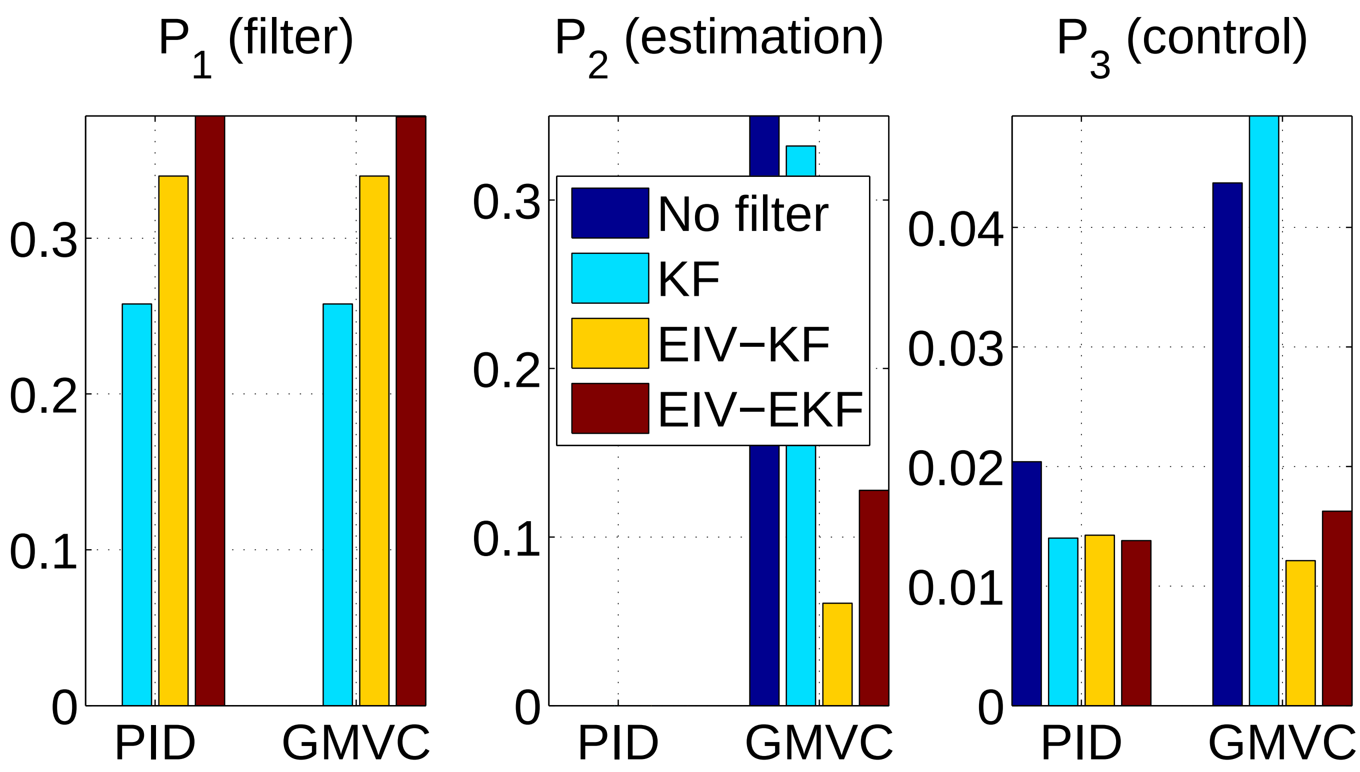
- 20 Monte-Carlo iterations with $N = 500$ samples
- Noise setup: $$E\left[\begin{bmatrix}\tilde{y}(k)\\ \tilde{u}(k-1)\end{bmatrix}\begin{bmatrix}\tilde{y}(k) & \tilde{u}(k-1)\end{bmatrix}\right] = \begin{bmatrix}0.50 & 0.30\\ 0.30 & 0.40\end{bmatrix} \tag{5}$$
- Reference signal: low-pass interpolated white noise:



- Performance indices:
  1. Filter performance: $P_1 = \left(\|w_0 - w\|_F - \|w_0 - \hat{w}_0\|_F\right) / \|w_0 - w\|_F$
  2. Estimation performance: $P_2 = 1/N \sum_{k=1}^{N} ||\theta - \hat{\theta}(k)||_2^2$
  3. Control performance: $P_3 = 1/N \sum_{k=1}^{N} ||r(k) - y_0(k)||_2^2$

Example 1: LTI SISO system

- LTI system: $$y(k) = 1.20y(k-1) - 0.81y(k-2) + 0.27y(k-3)$$ $$0.10u(k-1) + 0.17u(k-2) + 0.08u(k-3) \quad (6)$$
- True system parameters and noise covariance matrix assumed known

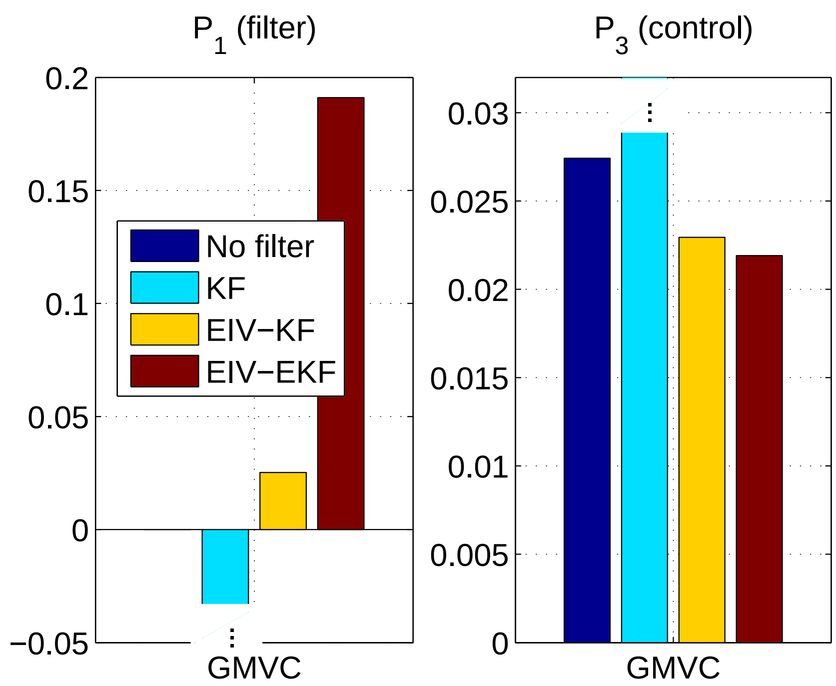


Example 2: Nonlinear SISO system

- Nonlinear system:

$$\begin{aligned} y(k) =& 1.20y(k-1) - 0.81y(k-2) + 0.27y(k-3) + \\ & 0.10u(k-1) + 0.17u(k-2) + 0.08u(k-3) + \\ & 0.08u(k-1)y(k-1) + 0.05u^2(k-1) \end{aligned} \quad (7)$$

- Default model: dynamical Frisch scheme off-line estimate



Conclusions & further work

Conclusions:

- Modification of the EIV setup for control purposes
- Potential benefits of EIV filtering techniques in terms of control performance for linear and nonlinear case

Further work:

- Use of estimated noise covariances matrix
- Joint EIV filtering and identification (*e.g.* Particle filters, Recursive Total Least Squares, Recursive Frisch scheme)
- Application to real plant through our industrial collaborators (Jaguar cars, Converteam Limited, Rolls Royce, ...)